MAT 681 HOMEWORK: FINITE FIELDS

1. With $\mathbb{F}_{11} = \{0, 1, 2, \ldots, 10\}$, compute the following.
   (a) $8 + 7$
   (b) 8×7
   (c) 8^7
   (d) 5^{-1}

2. Explain why it is not possible to represent $\mathbb{F}_9$ as $\{0, 1, 2, \ldots, 8\}$.

3. Explain how we know that $f(x) = x^2 + x + 1$ is not irreducible modulo 3, while $f(x) = x^2 + 1$ is.

4. With $\mathbb{F}_9 = \{0, 1, 2, x, x+1, x+2, 2x, 2x+1, 2x+2\}$ and $x^2 + 1 \equiv 0$, compute the following.
   (a) the characteristic of $\mathbb{F}_9$
   (b) $x^2 + 2x + 2$
   (c) 2^2
   (d) $(x+1)(x+2)$
   (e) $(x+1)^{-1}$
   (f) x^{-1}
   (g) $(x+1)^3$
   (h) $(x+a)^{3b}$, where $a \in \{0, 1, 2\}$ and $b \in \mathbb{N}$