MAT 681 ASSIGNMENT 3

Solve the following problems. Due in class Thursday, 15th September 2016.

1. Sometimes we want to use a monomial ordering that guarantees one monomial is larger than another. For instance, in the system $\{x^2z+y^3-4z^3, xyz-z^3\}$ it is very advantageous to have y^3 as the leading monomial of the first polynomial, and z^3 as the leading monomial of the second. The orderings you've studied so far won't guarantee this, but it's possible that a weighted ordering would. This exercise explores how to find such an ordering using systems of linear inequalities. (You'll get to brush up on simplex!)
   (a) Show that the entries of an admissible weighted ordering cannot be negative.
   Hint: If even one entry of a weighted ordering is negative, you can find a particular monomial t that will be smaller than another particular monomial u, contradicting one of the properties of admissible orderings.
   (b) Let a, b, c be indeterminates that represent the entries of a weighted ordering on x, y, and z. State a system of linear inequalities in a, b, and c that, *if consistent,* would guarantee the leading monomials we desire.
   (b) Use the simplex algorithm to determine whether your system is in fact consistent.
   Hint: A naïve system would have $f(a,b,c) > 0$. To use simplex, you need inequalities in the form $g(a,b,c) \geq d$. You can rewrite the former as the latter by introducing a particular $\varepsilon > 0$ and considering $f(a,b,c) \geq \varepsilon$. You can make ε small, but it's probably easier to use a "nice" number.
   Note: Yes, I really want you to do this with simplex.
   (c) From the point of view of computing a Gröbner basis, it would preferable to use an ordering with y^3 and z^2 as leading monomials because the S-polynomial of these two will reduce to zero modulo these two. *Prove this* by finding a syzygy on the two polynomials.
   Hint: Take inspiration from a previous homework's syzygy.

2. CCA, p. 24, Exercises 1, 3, 4, 7, 8

3. From *Peering into Advanced Mathematics*, all the True/False and Multiple Choice exercises on pp. 46–49, as well as Short Answer #4 on p. 49.