Plan

Prime Obsession

Goals:
(1) The students will find a math joke someone told me back in March funny.
(2) 360-ish pages between June 19th and July 18th, omitting the days of July 3rd and 5th. Three nominal weeks, but a fourth week when I am out of town provides time to catch up. This requires us to cover roughly 90 pages a week; or, roughly 21 pages a day.

- Week 1: By Tuesday, June 26th, read first five chapters (through p. 81).
- Week 2: No meeting.
- Week 3: By Tuesday, July 10th, read next ten chapters (through p. 264, 183 pages).
- Week 4: By Tuesday, July 17th, complete the text (through p. 361, 97 pages).

I would highly recommend that you read faster if you can. (I have now read the book twice, each time in no more than a week. So, it is possible.) If, for whatever reason, you cannot complete the entire reading, make sure you at least familiarize yourself with the material highlighted below. However, it's best not to miss anything.

I will give some easy quizzes to make sure you're reading the material. Otherwise, your grade will be based on an essay written at the end of the reading.

Topics of importance:

Chapter 1: sequences, series, infinite series, sequences of partial sums, divergence, convergence, limits, what is "infinity"?, the four major subdivisions of mathematics

Chapter 2: youth of Bernhard Riemann, Carl Friedrich Gauss, Richard Dedekind

Chapter 3: prime numbers, proof that there are infinitely many primes, functions, arguments, values, domains, Derbyshire's definition of exponential and logarithmic functions, the Prime Number Theorem

Chapter 4: life of Gauss, his motto, his grade-school accomplishment, how he upset Legendre, life of Leonhard Euler, Euler's style vs. Gauss', the St. Petersburg Academy

Chapter 5: the Basel problem and its solution, open and closed forms of numbers, Euler's generalization of the Basel problem, how to compute irrational powers of numbers, meaning of Σ, definition of the Riemann zeta function, real values of s for which $\zeta(s)$ definitely diverges

Chapter 6: counting vs. measuring, "the ghosts of departed quantities", analytic number theory, Zeno's paradox, connection between measuring and continuity, Lejeune Dirichlet, the rise of German mathematics, the Mendelssohn-mathematics connection, Dirichlet's theorem

Chapter 7: the Sieve of Eratosthenes, the Golden Key and its derivation, the gradient and the integral, $Li(N)$, the improved Prime Number Theorem

Chapter 8: Chebyshev's two results, Riemann's habilitation, non-Euclidean geometry, tragedy and nervous breakdown, appointment to Berlin Academy

Chapter 9: rewriting an infinite series as a finite expression, rewriting $\zeta(s)$ using $\eta(s)$, formula for $\zeta(1-s)$ in terms of $\zeta(s)$, trivial ("obvious") zeros of $\zeta(s)$ from the formula for $\zeta(1-s)$, full factorial function ($\gamma(x)$)

Chapter 10: the two components of a mathematical personality, Weierstrauss vs. Riemann, landmarks on the road to a proof of the Prime Number Theorem, the advantage of Hadamard's proof in comparison to de la Vallé Poisson's, the "weak Riemann

Hypothesis", the Hadamard-Disney connection, Stieltjes' "lost proof" of the Riemann Hypothesis

Chapter 11: Nine Zulu Queens Ruled China, the notions of natural, integer, rational, real, and complex numbers, algebraic and transcendental numbers, completeness of a number system, real and imaginary parts of complex numbers, complex arithmetic, density of rationals and irrationals, the complex plane, different sizes of infinity, modulus of a complex number, amplitude of a complex number, complex conjugate

Chapter 12: Hilbert and his personality, Kronecker and Gordan, stories of Hilber, Hilbert's eighth problem, the critical strip and the critical line, other challenging problems, the turn of the 20^{th} century, Gram's list of zeros

Chapter 13: one of the most beautiful identities in mathematics, how to define complex values for e, how e^x and $\ln x$ behave differently for complex numbers, how Riemann visualized complex functions (or at least x^2), the argument ant, the value ant

Chapter 14: British mathematics of the 19^{th} century, Hardy, Littlewood, Landau, Landau's comment on Noether, Hardy's 1914 result, Littlewood's 1914 result, Skewes' number, Von Koch's 1901 result

Chapter 15: Definition of Big Oh, mathematicians' sense of humor, the Mobius function, Mertens's function

Chapter 16: *Ignoramus et ignorabimus, wir müssen wissen, wir werden wissen,* Göttingen under the Nazis, our wrong idea of Nazis, verification of the Riemann Hypothesis without explicit computation of zeros, Alan Turing, Siegel's astonishing discovery

Chapter 17: Fields, finite fields, matrices, characteristic polynomials, eigenvalues, the trace of a matrix, Hermitian matrices, the Hilbert-Polya conjecture

Chapter 18: Repulsion of Hermitian eigenvalues, pair correlation of zeros for Riemann zeta function and eigenvalues of Hermitian matrices, the Montgomery-Odlyzka Law

Chapter 19: the J function, π in terms of μ and J, the Golden Key (Calculus Version)

Chapter 20: a "Riemann operator", relation to chaos, p-adic completion of $\mathbf{Q}$, Connes's adelic space, reviews of Connes's adelic space by mathematicians, Hejhal's "non-complex" preservation of the RH, the Cramér model and its problems, the law-court model of truth

Chapter 21: the secondary term of J, its relation to the roots of the Riemann zeta function, entire functions, raising a complex number to a complex exponent, why $\Sigma_p \mathrm{Li}(p)$ converges, "the periodic term"

Chapters 22 and Epilogue: no special topics

Questions to ponder

- Why is the Riemann conjecture important?
- Derbyshire mentions a number of difficult mathematics problems that long remained unsolved, such as Fermat's Last Theorem and the Goldbach Conjecture. Does he consider the Riemann Hypothesis to be more important? Why or why not?
- Do you agree with Derbyshire's explanation of counting numbers vs. measuring numbers?
- Derbyshire admits that he passes over a number of details, so as not to detract from the main point. When does the tradeoff between accuracy and accessibility become too strong in one direction or the other? Think about this especially when he deals with topics you should understand from your own background, such as conditional convergence, integrals around asymptotes, and matrices.
- On the other hand, Derbyshire's narrative includes many, many more mathematical explanations than does Gleick's. Which approach do you find more readable? Which approach do you find more informative? Which approach do you think is better for a wider audience?

- Derbyshire's presentation of scientific progress does not seem to agree with Gleick's. In particular, Derbyshire's recounting of the Riemann Hypothesis *generally* lacks dramatic personalities who fight an established, benighted order wedded to old ways of doing things. Do you think this is due more to differences between Derbyshire and Gleick, or to differences between mathematics and the other sciences?
- If someone asked you to describe the Riemann conjecture "in plain words", how would you go about trying?