Plan

The Pea and the Sun

Goals:
(1) The students will understand several paradoxes at the foundations of mathematics, and (a) how mathematicians resolve them, and/or (b) why mathematicians generally feel safe ignoring them.
(2) 170-ish pages between June 19th and July 18th, omitting the days of July 3rd and 5th. Eight class meetings. Three nominal weeks, but a fourth week when I am out of town provides time to catch up. This requires us to cover roughly 40 pages a week; or, roughly 6 pages a day.

- Week 1: By Thursday, June 28th, read first chapter (through p. 48).
- Week 2: No meeting.
- Week 3: By Thursday, July 12th, read next three chapters (through p. 142, 94 pages).
- Week 4: By Thursday, July 19th, complete the reading (through p. 172, 30 pages).

I've tried to time the readings on Thursdays so that you can spend the weekends on *Prime Obsession*, which requires more attention and time. Hopefully, you can spend more time reading *The Pea and the Sun* between Tuesday and Thursday.

I will give some easy quizzes to make sure you're reading the material. Otherwise, your grade will be based on an essay written at the end of the reading.

Topics of importance:

Chapter 1: different philosophies of mathematical foundations (Platonism, constructivism, formalism), infinity and cardinality, Cantor's definition of an infinite set, countable and dense sets, different sizes of infinity, the Continuum Hypothesis, well-ordered sets, the Axiom of Choice, ZF vs. ZFC, Banach, the Hausdorff paradox, Tarski, Gödel, undecidable statements and the Universal Truth Machine, Cohen, Euclid's fifth postulate, non-Euclidean geometries and their realities

Chapter 2: the three types of paradox, Braess's Paradox, Simpson's Paradox, Russell's Paradox, the Barber Paradox, self-referential statements, vanishing puzzles and counterfeitings

Chapter 3: sets, cardinality (again), basic operations of sets, why $c = 2^{\aleph_0}$, (2 to the aleph-nought) Cantor's Theorem, the Burali-Forti paradox, Cantor's Paradox, isometries, groups, the Wallace-Bolyai-Gerwein Theorem, the problem of equidecomposability, shifting to and from infinity

Chapter 4: Cantor's definition of an infinite set (again), injection/one-to-one, surjection/onto, bijection, Dedekind's definition of an infinite set, equivalence of the definitions, shiftin to and from infinity on a sphere, basic transfinite arithmetic, equality of content vs. equality of length, points as locations, a line segment and a square region, continuous vs. discontinuous maps, Lebesgue measure, Cantor dust, Sierpinski's carpet and sponge, the Vitali set and its paradoxes, Stewart's Hyperwebster, the Sierpinski-Mazurkiewicz Paradox

Chapter 5: statement of the Theorem, a group *G* of rotations, poles, orbit, the choice set *C*, the Hausdorff Paradox on the sphere, cutting templates

Chapter 6: reasons not to reject the Axiom of Choice, “paradoxes” in physics, whether the constructions are Lebesgue measurable, the views of Feynman and Hardy on the nature of mathematics

Chapter 7: Arlo Lipof's matter fabricator, Augenstein's speculation, “Gamow”'s Big Bang Theory, Svozil's origins for chaos, the electron-muon puzzle, a denial of reality?

Questions to ponder

- Is Hermann Weyl a Platonist? (see p. 30)
- What is the relationship between “Cows in the Maze” (the game, not the book) and Russell's Paradox (or the Barber's Paradox)?
- Do you see *R* in τ on p. 84?
- Wapner lists examples of elements that are in equivalence classes with respect to Vitali construction. Give several others.
- Describe an example Vitali Construction *M*.
- Why is the intersection of the sets *Mr* such that *r* is rational equal to the real numbers?
- Can you find a way to partition the integers *Z* into two sets *E*1 and *E*2 such that *Z* is isomorphic/congruent/equivalent to both *E*1 and *E*2?
- Do the paradoxes presented in Chapter 4 bother you as much as the Banach-Tarski paradox?
- Read the proof of the theorem carefully. Do you find it clear and convincing, or has Wapner missed the mark?
- Does Wapner's chapter on resolving the paradox suggest to you that he is a Platonist, a constructivist, or a formalist?
- Wapner suggests that Hardy's description of his work as useless is too harsh. Do you agree with Hardy or Wapner?
- Suppose we do discard the Axiom of Choice. Why would this not help much, even if it eliminates the Banach-Tarski Paradox?
- Does Banach-Tarski's relation to *immaterial* “points” or “locations” suggest that the problem is not with the mathematics, but with our interpretation of it as a model of the *material* world?