Modern Algebra 1 Section 1 · Assignment 5

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Exercise 1. *(pg. 53 Exercise 1) Find the gcd of the polynomials* $x^6+x^5-2x^4-x^3-x^2+2x$ *and* $3x^6+4x^5-3x^3-4x^2$ *in* $\mathbb{Q}[x]$*, and express them as* $af+bg$*, for some polynomials* a, b*.*

Solution:

Apply Euclid's algorithm to obtain

$$\begin{aligned} 3x^6+4x^5-3x^3-4x^2 &= 3\left(x^6+x^5-2x^4-x^3-x^2+2x\right)+\left(x^5+6x^4-x^2-6x\right) \\ x^6+x^5-2x^4-x^3-x^2+2x &= (x-5)\left(x^5+6x^4-x^2-6x\right)+\left(28x^4-28x\right) \\ x^5+6x^4-x^2-6x &= \frac{1}{28}(x+6)\left(28x^4-28x\right)+0. \end{aligned}$$

So the greatest common divisor is $28x^4-28x$, or $28x\left(x^3-1\right)$.

For the GCD identity,

$$\begin{aligned} 28x^4-28x &= \left(x^6+x^5-2x^4-x^3-x^2+2x\right)-(x-5)\left(x^5+6x^4-x^2-6x\right) \\ &= \left(x^6+x^5-2x^4-x^3-x^2+2x\right) \\ &\quad -(x-5)\left(\left(3x^6+4x^5-3x^3-4x^2\right)-3\left(x^6+x^5-2x^4-x^3-x^2+2x\right)\right) \\ &= (1+3(x-5))\left(x^6+x^5-2x^4-x^3-x^2+2x\right) \\ &\quad -(x-5)\left(3x^6+4x^5-3x^3-4x^2\right) \\ &= (3x-14)\left(x^6+x^5-2x^4-x^3-x^2+2x\right) \\ &\quad -(x-5)\left(3x^6+4x^5-3x^3-4x^2\right). \end{aligned}$$

So $a=3x-14$ and $b=-(x-5)$. ◊