CHAPTER 3 HOMEWORK, PART 1 (§§ 1–3)

MAT 421: NUMBER THEORY

Directions: Each group is responsible for all of the problems listed. No problem should be attempted before we cover the material indicated with it. I only need one submission from each group. I will give time in class for groups to meet and work; however, you should plan to meet outside class as well.

1. GROUPS

Group 1	Group 2	Group 3
Melissa Dyess	Aaron Ayers	Sr. Maria Acosta
Joel Huber	Kristie West	Lorelei Jones
Shannon West	Ryan Anderson	Stephanie Williams

2. EXERCISES

§3.1: Prime Numbers.

- *After the definition of prime, composite numbers:* p. 76 #2, 6, 14
  Hint on #6: Use the same approach that we used in class on #5.
- *After Theorem 3.1:* p. 76 #8, 10, 12
- *After the Sieve of Eratosthenes:* p. 76 #4
- *After Theorem 3.3:* p. 76 #16, 18, 20

§3.2: The Distribution of Primes.

- *After Theorem 3.5:* p. 90 #2
- *After Bertrand's Postulate:* p. 90 #6, 28
  Hint on #28: Use induction.
- *After the Twin Prime Conjecture:* p. 90 #4
- *After Goldbach's Conjecture:* p. 90 #12
- *After the Legendre Conjecture:* p. 90 #8

§3.3: Greatest Common Divisors and Their Properties.

- *Ab ovo:* p. 99 #2, 4, 6, 10
  Hint on #6: Use the Division Theorem for primes larger than 2.
  Hint on #10: Rewrite $(a+b)+(a-b)$ *and* $(a+b)-(a-b)$ *two different ways, one of them using* $\gcd(a+b, a-b)$.
- *After Theorem 3.7:* p. 99 #16, 24
- *After Lemma 3.2:* p. 100 #22
  Hint on #22: Use induction, Theorem 3.8, and Lemma 3.2.
- *After the definition of mutually relatively prime and pairwise relatively prime:* p. 99 #18

i	1	2	3	4	5	6	7	8
$x = 10^i$	10	100	1000	10,000	100,000	1,000,000	10,000,000	100,000,000
$\pi(x)$	4	25	168	1,229	9,592	78,498	664,579	5,761,455
$\frac{x}{\pi(x)}$	2.5	4	5.95	8.14	10.43	12.74	15.05	17.36

Table 1

Question 1. *Do you see a long-term pattern to* $\frac{x}{\pi(x)}$*?*

i	1	2	3	4	5	6	7	8	9
$x = e^i$	e	e^2	e^3	e^4	e^5	e^6	e^7	e^8	e^9
$\pi(x)$	1	4	8	16	34	79	183	429	1,019
$\frac{x}{\pi(x)}$	e	1.85	2.51	3.41	4.37	5.11	6.00	6.95	7.95

i	10	11	12	13	14	15
$x = e^i$	e^{10}	e^{11}	e^{12}	e^{13}	e^{14}	e^{15}
$\pi(x)$	2,466	6,048	14,912	37,128	93,117	234,855
$\frac{x}{\pi(x)}$	8.93	9.90	10.91	11.92	12.91	13.92

Table 2

Question 2. *Do you see a long-term pattern to* $\frac{x}{\pi(x)}$*?*