Programming Exercises

1. Implement the following pseudocode in Sage.
   procedure *Taylor4*
   inputs $a \in \mathbb{R}$
   f
   outputs
   the truncated Taylor series for $f(x)$ around $x = a$
   do
   let *result* $= f(a)$
   add $f'(a) \cdot (x - a)$ to *result*
   add $f''(a) \cdot (x - a)^2 / 2$ to *result*
   add $f'''(a) \cdot (x - a)^3 / 6$ to *result*
   add $f^{(4)}(a) \cdot (x - a)^4 / 24$ to *result*
   return *result*

2. Use your program to estimate the following numbers. Compare it to the "true" value by using Sage's `round()`. Indicate which results are within 1% of the correct answer.
   (a) $\sin 3$
   Hint: Use $f(x) = \sin x$ and $a = \pi$, then substitute 3 into the result of *Taylor4*
   (b) $\sqrt{3}$
   Hint: Use $f(x) = \sqrt{x}$ and $a = 4$, then substitute 3 into the result of *Taylor4*
   (c) $\ln 2$

3. Plot the result of *Taylor4* in part(a) with a dashed blue. Combine it with a plot of $\sin x$ with a black line, width 2.

4. You may have noticed some errors in the pseudocode's *formatting*: that is, it doesn't obey the format I described in class & in the textbook. Retype the pseudocode in your Sage worksheet, correcting any errors.

5. As usual, make sure your name is on the worksheet (in a *cell at the top* but *not in the title*); use sectioning, etc.