MAT 305: Mathematical Computing

Interactive worksheets in Sage

John Perry

University of Southern Mississippi

Spring 2019

Outline

You should be in worksheet mode to repeat the examples.

Outline

Interactive worksheets?

An *interactive worksheet* allows a user to visualize and manipulate concepts in a hands-on fashion.

- buttons, sliders, checkboxes
- graphics updated immediately or on demand

Creating interactive worksheets

"Procedure decorator": `@interact`

- Place immediately before definition of procedure
- Formal argument list consists of interact objects
  - input box
  - slider
  - checkbox
  - dropdown menu
  - buttons
  - color selector

Example

```
sage:  @interact
       def i_deriv(f=input_box(label='$f$')):
         if (f != None):
           print 'The derivative of ', f,
                 'is', diff(f)
```

Example

```
sage:  @interact
       def i_deriv(f=input_box(label='$f$')):
         if (f != None):
           print 'The derivative of ', f,
                 'is', diff(f)
```

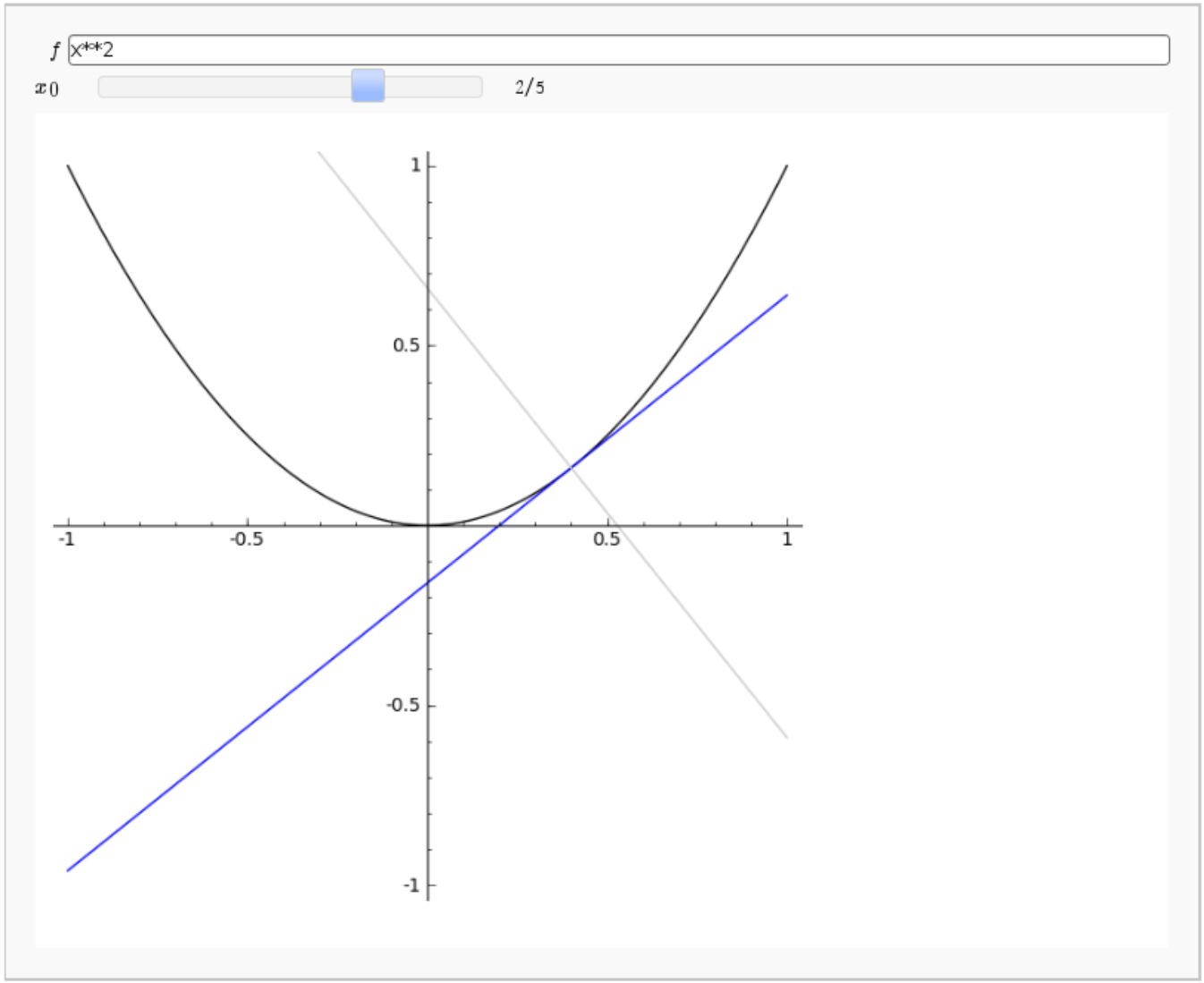
f x**5-3*x*cos(x)

```
The derivative of x^5 - 3*x*cos(x) is 5*x^4 + 3*x*sin(x) - 3*cos(x)
```

Something more visual

```
sage: xmin, xmax = -1, 1
sage: @interact
      def i_tan_norm(f=input_box(label='$f$'),
          x0=slider(xmin,xmax,label='$x_0$',
          step_size=1/10,default=0)):
        if f != None and f != '':
          y0 = f(x=x0)
          mtan = (diff(f))(x=x0)
          mnorm = -1/mtan
          fplot = plot(f,xmin,xmax,color='black')
          tan_plot = plot(mtan*(x-x0)+y0,xmin,xmax)
          norm_plot = plot(mnorm*(x-x0)+y0,xmin,
                      xmax,color=(0.8,0.8,0.8))
          show(fplot+tan_plot+norm_plot,ymin=-1,
               ymax=1,aspect_ratio=1)
```

. . . the result



Outline

Usage

- argument to interactive procedure

- *id* = *object*(*options*) where
  - *id* is an argument for the value of the object
  - *object* is one of the object commands given below
  - *options* specify object's properties
    - two options common to all objects

Command options for all objects

- `label` = *label*
  a string that labels the object
  - limited LaTeX
  - compare `label='x_0'`, `label='$x_0$'`

- `default` = *value*
  the default value of the object, if any

Text, numbers, functions, ...

`input_box(`*options*`)` where *options* include

- `width`: width of box (# letters)

User enters text (function, number, etc.)

Example

```
f = input_box(label='$f$', default=x*cos(x),
              width=10)
```

A number within an interval

`slider(`*options*`)` where *options* are

- continuous slider?
  - `vmin`: minimum value of slider
  - `vmax`: maximum value of slider
- discrete slider?
  - list of values: `vmin`, no `vmax`

  or
  - range and step size: `vmin`, `vmax`, `step_size`
- `display_value`:
  - `True`? show slider's value on right
  - `False`? don't

User slides knob across line to select value

Example

```
x0 = slider(label='$x_0$', vmin=-1, vmax=1,
            default=0, step_size=1/10)
```

A subinterval of an interval

`range_slider(`*options*`)` where *options* are those of `slider()`, but...

- `default` and *result* are be *a pair* of numbers

User slides *two* knobs across line to select subinterval

Example

```
x0 = slider(label='$x_0$', vmin=-10, vmax=10,
            default=(-1,1), step_size=1/10)
```

Switches or flags (on/off, yes/no)

`checkbox(`*options*`)`

- User sets boolean (on/off or `True`/`False`) value

Example

```
show_tangent = checkbox(label='show tangent',
                        default=True)
```

Colors

`Color(`*color definition*`)` where

- *color definition* is
  - a recognized name for a color
  - an rgb triplet
  - a hex string (don't worry about this one unless you already know what I mean)
- "common" options do not work with this object

User manipulates color using string, circle, box

Example

```
col = Color(0,0,1)
```

Outline

Example problem

Problem

Given f and c, plot

- f,
- its derivative df on an interval $[a, b]$, and
- a line tangent to f at $x = c \in [a, b]$.

Reuse old code

We already know how to compute line tangent to a function: (don't retype)

```
sage:  def tangent_line(f, a, x=x):
           # redefine f
           f(x) = x
           # point-slope form of a line
           b = f(x=a)
           df(x) = diff(f,x)
           m = df(a)
           result = m*(x - a) + b
           return result
```

Reuse old code

We already know how to compute line tangent to a function: (don't retype)

```
sage:  def tangent_line(f, a, x=x):
           # redefine f
           f(x) = x
           # point-slope form of a line
           b = f(x=a)
           df(x) = diff(f,x)
           m = df(a)
           result = m*(x - a) + b
           return result
```

Saved in `calc_utils.py`, so:

```
sage:  %attach calc_utils.sage
```

Interactive procedure

inputs

f, a function

$c \in \mathbb{R}$

do

plot f

plot f'

plot line tangent to f at $x = c$

Interactive procedure

inputs

f, a function

$c \in \mathbb{R}$

do

plot f

plot f'

plot line tangent to f at $x = c$

…over what interval? **refine pseudocode**

Interactive procedure

inputs

f, a function

$a, b \in \mathbb{R}$

$t \in [0, 1]$

do

plot f on $[a, b]$

plot f' on $[a, b]$

let $c = t(b - a)$

plot on $[a, b]$ line tangent to f at $x = c$

Interactive procedure

inputs

f, a function

$a, b \in \mathbb{R}$

$t \in [0, 1]$

do

plot f on $[a, b]$

plot f' on $[a, b]$

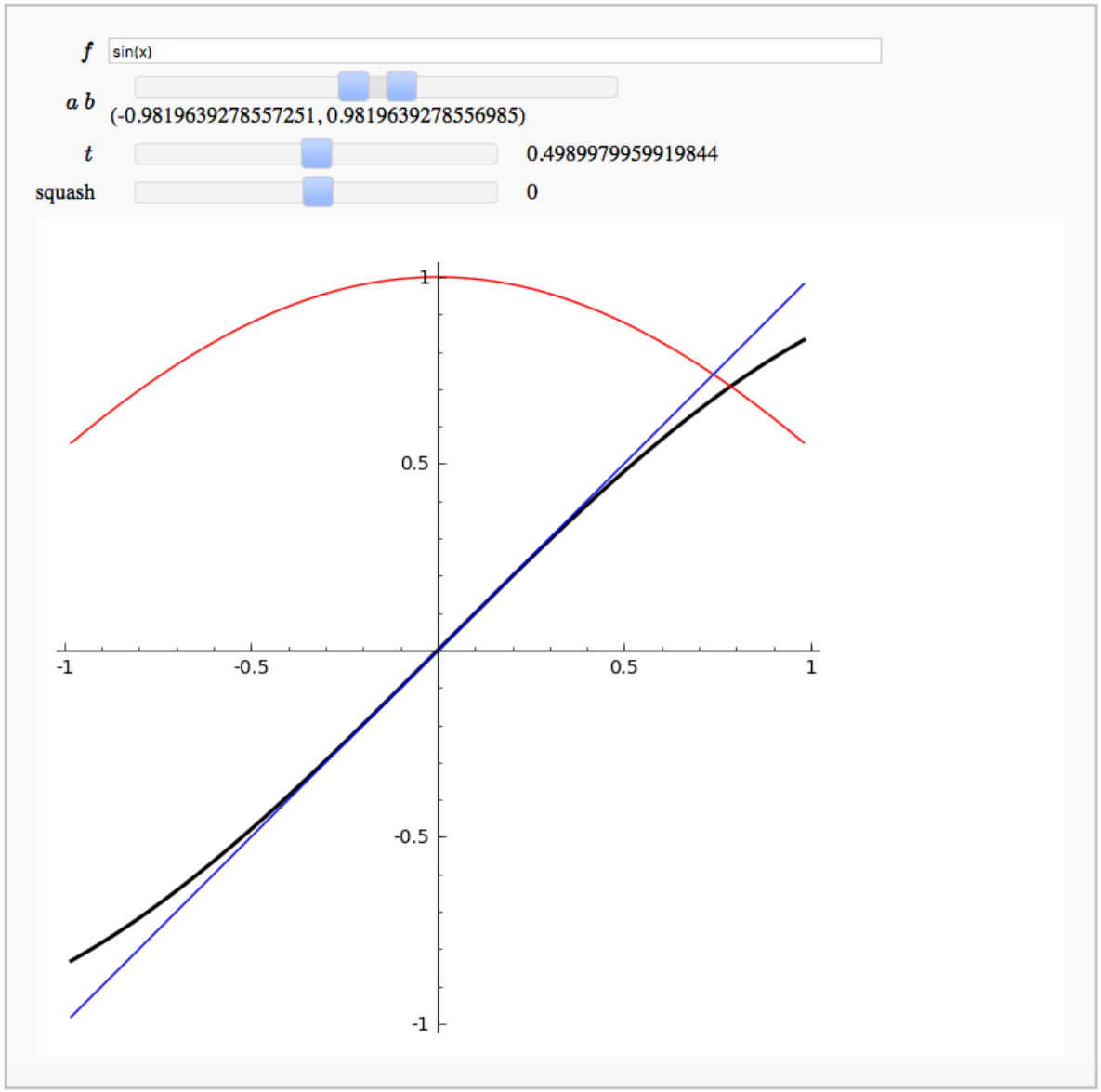
let $c = t(b - a)$ — parameterized c on $[a, b]$ using t

plot on $[a, b]$ line tangent to f at $x = c$

Sage code

```
sage: @interact
      def plot_f_df_tline(
            f=input_box(default=sin(x), label='$f$'),
            subint=range_slider(-10, 10, default=(-1,1),
                                label='$a\ b$')
            t=slider(0, 1, default=0.5, label='$t$'),
            w=slider(-19, 19, 1, default=0, label='squash')
      ):

          a, b = subint # subinterval on which to plot
          p = plot(f, a, b, color='black', thickness=2)

          # t tells us how far to move along [a,b]

          # 0 means a; 1 means b; other values proportional
          c = a + t*(b - a)

          df(x) = diff(f)
          p = p + plot(df, a, b, color='red')
          f(x) = f
          p = p + plot(df(c)*(x-c)+f(c), a, b)
          show(p, aspect_ratio=(20-w)/20)
```

Result


f
sin(x)
a b
(-0.9819639278557251, 0.9819639278556985)
t
0.4989979959919844
squash
0
1
0.5
-1
-0.5
0.5
1
-0.5
-1

Outline

Summary

- Interactive worksheets help user visualize, manipulate concepts
- Use `@interact` procedure decorator
- Several easy-to-define interface objects
- Break procedures into parts
  - easy to read
  - easy to reuse
  - easy to change