

MAT 305: Mathematical Computing

Procedures

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Outline

- 1 Procedures
- 2 Arguments to procedures
- 3 Extended example
- 4 Returning values
- 5 Continuing the example
- 6 Pseudocode
- 7 Scripting
- 8 Summary

Procedures

Arguments to
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Procedures?

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Pseudocode

Scripting

Summary

- sequence of statements organized as one command
 - may return one or more values
- other languages:

Ada, Pascal	procedure and function
BASIC	subroutine
C, C++, Kotlin, Python, VB	function
Eiffel	feature
Java	method

- We use “procedure” to avoid confusion w/mathematical functions

Why procedures?

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Summary

- avoid retyping code
 - many patterns repeated
 - same behavior, different data
- organization, abstraction
- easier to read, maintain

Defining a procedure

```
def name( argument1=default1 , argument2=default2 , ... ) :  
    statement1  
    statement2  
    ...
```

where

- *name* is an identifier
- *arguments* (optional) are identifiers
- *defaults* (optional) are default values for the corresponding arguments

Defining a procedure

```
def name( argument1=default1 , argument2=default2 , ... ) :  
    statement1  
    statement2  
    ...
```

where

- *name* is an identifier
- *arguments* (optional) are identifiers
- *defaults* (optional) are default values for the corresponding arguments

not optional:

- *:, (), def*
- *at least one statement*
- *indent all statements in procedure*

Calling a procedure

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Summary

- once p is defined, call using $p()$
- supply data for arguments without default values

Example

```
def greetings():  
    # greet the user  
    print 'Greetings!'
```

- name of procedure is greetings
- no arguments
- one statement
- one comment (# greet the user)
 - ignored by computer
 - useful for programmers

Example

Try it!

```
sage: def greetings():  
        # greet the user  
        print 'Greetings!'  
sage: greetings()  
Greetings!
```

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Arguments?

placeholders for data

scope: identifier valid only inside procedure where it is defined

- data still exists outside procedure
- modifying argument does not modify original data, but creates new data
 - *caveat:* contents of lists and sets can be modified
- value of data forgotten immediately after procedure concludes

Example

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Summary

```
def greetings(name='Leonhard Euler'):  
    print 'Greetings,', name
```

- name of procedure is greetings
- one argument, name
 - default value: 'Leonhard Euler'

Example

Try it!

```
sage: def greetings(name='Leonhard Euler'):  
      print 'Greetings,', name
```

```
sage: greetings()  
Greetings, Leonhard Euler
```

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Summary

Example

Try it!

```
sage: def greetings(name='Leonhard Euler'):  
      print 'Greetings,', name
```

```
sage: greetings()  
Greetings, Leonhard Euler
```

```
sage: greetings('Pythagoras')  
Greetings, Pythagoras
```

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Summary

Example

Try it!

```
sage: def greetings(name='Leonhard Euler'):
      print 'Greetings,', name
```

```
sage: greetings()
Greetings, Leonhard Euler
```

```
sage: greetings('Pythagoras')
Greetings, Pythagoras
```

```
sage: greetings(name='Pythagoras')
Greetings, Pythagoras
```

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Example

Try it!

```
sage: def greetings(name='Leonhard Euler'):  
      print 'Greetings,', name
```

```
sage: greetings()  
Greetings, Leonhard Euler
```

```
sage: greetings('Pythagoras')  
Greetings, Pythagoras
```

```
sage: greetings(name='Pythagoras')  
Greetings, Pythagoras
```

```
sage: greetings(pi)  
Greetings, pi
```

Warning 1

Procedures

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Summary

Don't use uninitialized identifiers

```
sage: def greetings(name='Leonhard Euler'):
        print 'Greetings,', name
```

```
sage: greetings(Pythagoras) oops: no quotes!
...Output deleted...
```

```
NameError: name 'Pythagoras' is not defined
```

Warning 2

Procedures

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Pseudocode

Scripting

Summary

Scope implies name does not exist outside hello

```
sage: def greetings(name='Leonhard Euler'):
      print 'Greetings,', name
```

```
sage: greetings('Pythagoras')
Hello, Pythagoras
```

```
sage: name
```

...Output deleted...

```
NameError: name 'name' is not defined
```

name out of scope

Warning 3

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Summary

Scope implies name forgotten once hello concludes

```
sage: def greetings(name='Leonhard Euler'):
      print 'Greetings,', name
```

```
sage: greetings('Pythagoras')
```

```
Hello, Pythagoras
```

```
sage: greetings()
```

```
Hello, Leonhard Euler
```

name has value 'Leonhard Euler' again

Warning 4

Can change value inside procedure, but value outside procedure remains the same

```
sage: def mischievous_hello(name='world'):  
      name = 'loser!'  
      print 'Hello,', name
```

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Warning 4

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Summary

Can change value inside procedure, but value outside procedure remains the same

```
sage: def mischievous_hello(name='world'):  
      name = 'loser!'  
      print 'Hello,', name
```

```
sage: print_name = 'Dr. Perry'
```

```
sage: mischievous_hello(print_name)
```

```
Hello, loser! value of name changed in procedure
```

Warning 4

Can change value inside procedure, but value outside procedure remains the same

```
sage: def mischievous_hello(name='world'):  
      name = 'loser!'  
      print 'Hello,', name
```

```
sage: print_name = 'Dr. Perry'
```

```
sage: mischievous_hello(print_name)
```

```
Hello, loser! value of name changed in procedure
```

```
sage: print_name
```

```
'Dr. Perry' value of print_name unchanged
```

Warning 5

Procedures

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Scripting

Summary

If defaults are not given to arguments, you must supply something

```
sage: def greetings(name):  
        print 'Greetings,', name
```

```
sage: greetings()
```

...Output deleted...

```
TypeError: greetings() takes exactly 1 argument (0  
given)
```


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A mathematical problem

Given function f and real number a
find line tangent to f at $x = a$

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Summary

A mathematical problem

Given function f and real number a
find line tangent to f at $x = a$

How should we implement this?

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Summary

A mathematical problem

Given function f and real number a
find line tangent to f at $x = a$

How should we implement this?

```
sage: def tangent_line(f, a):  
        # point-slope form of a line
```

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Summary

A mathematical problem

Given function f and real number a
find line tangent to f at $x = a$

How should we implement this?

```
sage: def tangent_line(f, a):  
        # point-slope form of a line  
        b = f(a)  
        df(x) = diff(f,x)  
        m = df(a)  
        result = m*(x - a) + b
```

A mathematical problem

Given function f and real number a
find line tangent to f at $x = a$

How should we implement this?

```
sage: def tangent_line(f, a):  
    # point-slope form of a line  
    b = f(a)  
    df(x) = diff(f,x)  
    m = df(a)  
    result = m*(x - a) + b  
    print 'The line tangent to', f,  
    print 'at x =', a, 'is',  
    print result
```

Quick test

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Summary

```
sage: f(x) = x^2
```

```
sage: tangent_line(x^2, 1)
```

The line tangent to $x \mapsto x^2$

at $x = 1$ is

```
2*x - 1
```

Trouble is...

...it sometimes complains!

```
sage: tangent_line(x^2, 1)
```

...*Output deleted*...

DeprecationWarning: Substitution using
function-call syntax and unnamed arguments is
deprecated and will be removed from a future release
of Sage; you can use named arguments instead, like
EXPR(x=..., y=...)

...*Output deleted*...

The line tangent to x^2 at $x = 1$ is $2*x - 1$

Trouble is...

...it sometimes complains!

```
sage: tangent_line(x^2, 1)
```

...*Output deleted*...

DeprecationWarning: Substitution using function-call syntax and unnamed arguments is deprecated and will be removed from a future release of Sage; you can use named arguments instead, like `EXPR(x=..., y=...)`

...*Output deleted*...

The line tangent to x^2 at $x = 1$ is $2*x - 1$

Problematic line: $b = f(a)$

- f has value x^2 , an *expression*
 - $b = f(a)$ wants f to be a *function*

Solution #1

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Summary

Define f as function, call `tangent_line()` w/ f

```
sage: f(x) = x^2
```

```
sage: tangent_line(f, 1)
```

The line tangent to $x \mapsto x^2$ at $x = 1$ is $2x - 1$

Solution #1

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Summary

Define f as function, call `tangent_line()` w/ f

```
sage: f(x) = x^2
```

```
sage: tangent_line(f, 1)
```

The line tangent to $x \mapsto x^2$ at $x = 1$ is $2x - 1$

Inconvenient, though: `tangent_line(x^2, 1)` would be useful.

Solution #2

Specify variable of substitution in procedure:

```
sage: def tangent_line(f, a):  
        # point-slope form of a line  
        b = f(x=a)  
        df(x) = diff(f,x)  
        ...
```

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Solution #2

Specify variable of substitution in procedure:

```
sage: def tangent_line(f, a):  
        # point-slope form of a line  
        b = f(x=a)  
        df(x) = diff(f,x)  
        ...
```

```
sage: tangent_line(x^2, 1)
```

The line tangent to $x \mapsto x^2$ at $x = 1$ is $2*x - 1$

Also undesirable. *Why?*

Solution #2

Specify variable of substitution in procedure:

```
sage: def tangent_line(f, a):  
        # point-slope form of a line  
        b = f(x=a)  
        df(x) = diff(f,x)  
        ...
```

```
sage: tangent_line(x^2, 1)
```

The line tangent to $x \mapsto x^2$ at $x = 1$ is $2*x - 1$

Also undesirable. *Why?*

```
sage: var('t')
```

```
sage: tangent_line(t^2, 1)
```

The line tangent to $x \mapsto t^2$ at $x = 1$ is t^2

Redefine expression as function:

```
sage: def tangent_line(f, a):  
      # redefine f  
      f(x) = f  
      # point-slope form of a line  
      b = f(a)  
      ...
```

Solution #3

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Summary

Redefine expression as function:

```
sage: def tangent_line(f, a):  
        # redefine f  
        f(x) = f  
        # point-slope form of a line  
        b = f(a)  
        ...
```

```
sage: tangent_line(x^2, 1)
```

The line tangent to $t \mapsto t^2$ at $x = 1$ is $2*x - 1$

Better, but insufficient. *Why?*

Solution #3

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Summary

Redefine expression as function:

```
sage: def tangent_line(f, a):  
        # redefine f  
        f(x) = f  
        # point-slope form of a line  
        b = f(a)  
        ...
```

```
sage: tangent_line(x^2, 1)
```

The line tangent to $t \mapsto t^2$ at $x = 1$ is $2*x - 1$

Better, but insufficient. *Why?*

Why should t change to x ?

Best (?) solution

Redefine as function in indeterminate:

```
sage: def tangent_line(f, a, x=x):  
      # redefine f  
      f(x) = f  
      # point-slope form of a line  
      b = f(a)  
      df(x) = diff(f,x)  
      m = df(a)  
      result = m*(x - a) + b  
      print 'The line tangent to', f,  
      print 'at', x, '=', a, 'is',  
      print result
```

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Summary

Best (?) solution

Redefine as function in indeterminate:

```
sage: def tangent_line(f, a, x=x):  
    # redefine f  
    f(x) = f  
    # point-slope form of a line  
    b = f(a)  
    df(x) = diff(f,x)  
    m = df(a)  
    result = m*(x - a) + b  
    print 'The line tangent to', f,  
    print 'at', x, '=', a, 'is',  
    print result  
  
sage: tangent_line(t^2, 1)
```

The line tangent to $t \mapsto t^2$ at $t = 1$ is $2*t - 1$

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Returning values?

- procedures compute values
- Often want to work with what we've computed
 - print command unhelpful
 - copy and paste annoying

Example

Derivative necessary to compute many things:

- graph tangent line
- analyze concavity
- identify optimum values
- ...

The return command

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Summary

`return` *value1*, *value2*, ...

- reports the data values *value1*, *value2*, etc. to caller
- only works inside procedures

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Write a procedure whose arguments are

- a function f ,
- numbers $[a, b]$,
- a number $c \in (a, b)$,
- an indeterminate x ,

and returns both f and the line tangent to it at $x = c$ on $[a, b]$.

Divide and conquer

How can we break this into smaller tasks?

Divide and conquer

How can we break this into smaller tasks?

- 1 Compute line tangent to f at c
- 2 Plot f on $[a, b]$
- 3 Plot line on $[a, b]$
- 4 Return their sum

Divide and conquer

How can we break this into smaller tasks?

- 1 Compute line tangent to f at c
- 2 Plot f on $[a, b]$
- 3 Plot line on $[a, b]$
- 4 Return their sum

already solved

Tangent line: modify implementation

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Summary

```
sage: def tangent_line(f, a, x=x):  
      # redefine f  
      f(x) = x  
      # point-slope form of a line  
      b = f(x=a)  
      df(x) = diff(f,x)  
      m = df(a)  
      result = m*(x - a) + b  
      return result
```

Tangent line: modify implementation

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Summary

```
sage: def tangent_line(f, a, x=x):
        # redefine f
        f(x) = x
        # point-slope form of a line
        b = f(x=a)
        df(x) = diff(f,x)
        m = df(a)
        result = m*(x - a) + b
        return result
sage: tangent_line(t^2, 1, t)
2*t - 1
```

Use to plot

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Summary

```
sage: def tangent_plot(f, a, b, c, x=x):
```

Use to plot

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Summary

```
sage: def tangent_plot(f, a, b, c, x=x):  
      # compute the line
```

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Summary

```
sage: def tangent_plot(f, a, b, c, x=x):  
      # compute the line  
      tan_line = tangent_line(f, c, x)
```


Use to plot

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Summary

```
sage: def tangent_plot(f, a, b, c, x=x):  
        # compute the line  
        tan_line = tangent_line(f, c, x)  
        # plot of f
```

Use to plot

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Summary

```
sage: def tangent_plot(f, a, b, c, x=x):  
      # compute the line  
      tan_line = tangent_line(f, c, x)  
      # plot of f  
      f_plot = plot(f, a, b)
```

Use to plot

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Pseudocode

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Summary

```
sage: def tangent_plot(f, a, b, c, x=x):  
      # compute the line  
      tan_line = tangent_line(f, c, x)  
      # plot of f  
      f_plot = plot(f, a, b)  
      # plot of line
```

Use to plot

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Summary

```
sage: def tangent_plot(f, a, b, c, x=x):  
        # compute the line  
        tan_line = tangent_line(f, c, x)  
        # plot of f  
        f_plot = plot(f, a, b)  
        # plot of line  
        tan_plot = plot(tan_line, a, b)
```

Use to plot

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Summary

```
sage: def tangent_plot(f, a, b, c, x=x):  
      # compute the line  
      tan_line = tangent_line(f, c, x)  
      # plot of f  
      f_plot = plot(f, a, b)  
      # plot of line  
      tan_plot = plot(tan_line, a, b)  
      # combine & return
```

Use to plot

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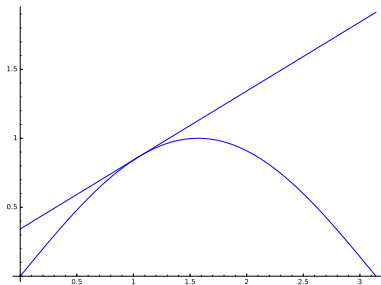
Pseudocode

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Summary

```
sage: def tangent_plot(f, a, b, c, x=x):  
      # compute the line  
      tan_line = tangent_line(f, c, x)  
      # plot of f  
      f_plot = plot(f, a, b)  
      # plot of line  
      tan_plot = plot(tan_line, a, b)  
      # combine & return  
      return f_plot + tan_plot
```

```
sage: tangent_plot(sin(x), 0, pi, pi/3)
```



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Pseudocode?

description of algorithm

- many formats
- format independent of computer language
- prefer mathematics to programming
 - “ i th element of L ” or “ L_i ”, not $L[i-1]$

Our pseudocode format

algorithm *name*

inputs

input1 \in *domain1*

input2, *description of type*

...

outputs

output1, *relationship to inputs*

output2, *relationship to inputs*

...

do

English or mathematical statement 1

English or mathematical statement 2

...

Our pseudocode format

algorithm *name*

inputs

input1 \in *domain1*

input2, *description of type*

...

outputs

output1, *relationship to inputs*

output2, *relationship to inputs*

...

do

English or mathematical statement 1

English or mathematical statement 2

...

Try it now on the given problem

Example problem

Write a program to compute the line tangent to $f(x)$ at $x = x_0$.

- Write pseudocode answering:
 - ① What inputs will we need?
 - domain of each input (what set/type of object)
 - ② What outputs do we expect?
 - inputs' purpose & relationship to output
 - ③ How do we use the inputs to generate the output?
 - think step-by-step
 - do a sample problem: $f(x) = x^2$, $x_0 = 3$
 - think about possible errors errors
- Implement pseudocode

Example pseudocode

algorithm *tangent_line*

inputs

f , a function of a variable x

$x_0 \in \mathbb{R}$

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Example pseudocode

algorithm *tangent_line*

inputs

f , a function of a variable x

$x_0 \in \mathbb{R}$

outputs

the line tangent to $f(x)$ at $x = x_0$

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Example pseudocode

algorithm *tangent_line*

inputs

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do

— *We need two things for a line: a point (x_0, y_0) and the slope m*

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Let $y_0 = f(x_0)$

– *Use Calculus to find m*

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Let $fderiv = f'(x)$

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– *Point-slope form: $y - y_0 = m(x - x_0)$*

Let $line = m(x - x_0) + y_0$

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– *Point-slope form: $y - y_0 = m(x - x_0)$*

Let $line = m(x - x_0) + y_0$

return *line*

Example implementation

Compare to implementation:

```
def tangent_line(f, a, x=x):  
    # redefine f  
    f(x) = x  
    # point-slope form of a line  
    b = f(x=a)  
    df(x) = diff(f,x)  
    m = df(a)  
    result = m*(x - a) + b  
    return result
```

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    m = df(a)  
    result = m*(x - a) + b  
    return result
```

Specify x as input to pseudocode?

- not inherent to *problem*, but to *programming*, so
- do not include in pseudocode, but
- this is arguable

Example run

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Behold: the power of symbolic computation!

```
sage: var('a b c')
```

```
sage: tangent_line(a*x^2 + b*x + c, 1, x)  
(2*a + b)*(x - 1) + a + b + c
```

Combine with plots

We show the plots of e^x and its tangent line at $x = 0$

```
sage: f = e^t
sage: tanline = tangent_line(f, 0, t)
sage: fplot = plot(f, -2, 2, color='black',
                  thickness=2)
sage: lineplot = plot(tanline, -2, 2,
                     linestyle='dashed')
sage: fplot + lineplot
```

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Combine with plots

We show the plots of e^x and its tangent line at $x = 0$

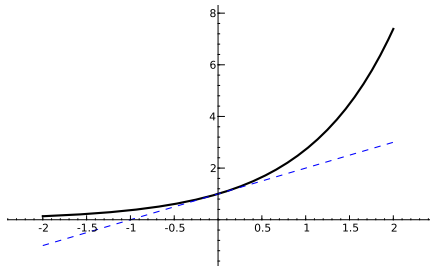
```
sage: f = e^t
```

```
sage: tanline = tangent_line(f, 0, t)
```

```
sage: fplot = plot(f, -2, 2, color='black',  
                  thickness=2)
```

```
sage: lineplot = plot(tanline, -2, 2,  
                     linestyle='dashed')
```

```
sage: fplot + lineplot
```



Combining procedures

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Summary

It would be nice to have a procedure that graphs an arbitrary $f(x)$ and its tangent line at $x = x_0$. Options include:

Combining procedures

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Summary

It would be nice to have a procedure that graphs an arbitrary $f(x)$ and its tangent line at $x = x_0$. Options include:

- Repeat previous commands for each f and each x_0

a lot of work!

Combining procedures

It would be nice to have a procedure that graphs an arbitrary $f(x)$ and its tangent line at $x = x_0$. Options include:

- Repeat previous commands for each f and each x_0
a lot of work!
- Encapsulate commands in another procedure

Pseudocode

algorithm *plot_function_and_tangent*

inputs

f , a function of a variable x

$x_0 \in \mathbb{R}$

outputs

the plot of $f(x)$ and the line tangent to f at $x = x_0$

do

Let P_1 be the plot of $f(x)$ in a neighborhood of x_0

Let $g(x)$ be the line tangent to f at x_0

Already solved!

Let P_2 be the plot of $g(x)$ in the same neighborhood of x_0

return P_1 and P_2 combined

Implementation

```
def plot_function_and_tangent(f, x0, x=x,  
                             xmin=-2, xmax=2):  
  
    # plots f(x) and line tangent to f at x0  
    # over [ xmin, xmax ];  
    # returns combination of these plots  
  
    P1 = plot(f, xmin, xmax, rgbcolor='black',  
             thickness=2)  
  
    # next line reuses previous code  
    g = tangent_line(f, x0, x)  
  
    P2 = plot(g, xmin, xmax, linestyle='dashed')  
    return P1 + P2
```

Whitespace

distinguishes

different

tasks

Examples

```
sage: def plot_function_and_tangent...
```

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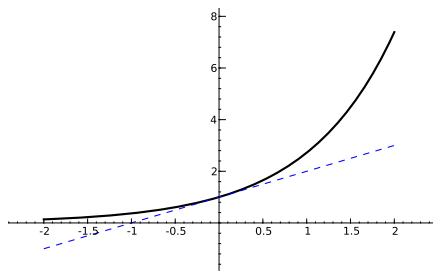
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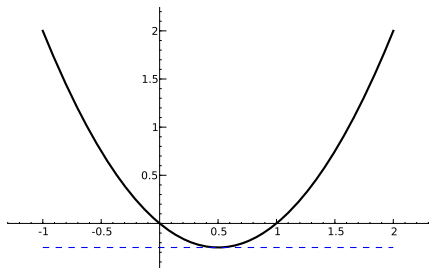
```
sage: def plot_function_and_tangent...  
sage: plot_function_and_tangent(e^x,x)
```



- Procedures
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- Extended example
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Examples

```
sage: def plot_function_and_tangent...  
sage: plot_function_and_tangent(x^2-x,0.5,xmin=-1,  
                                xmax=2)
```



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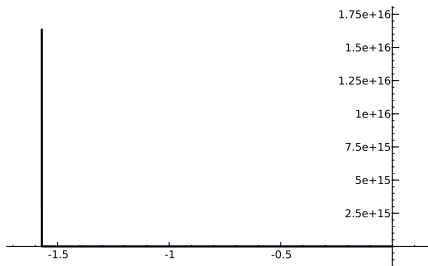
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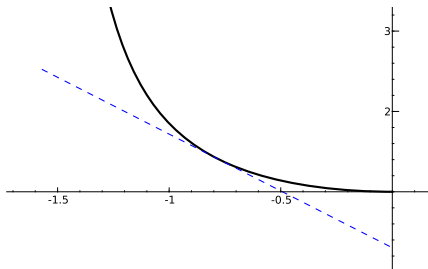
```
sage: def plot_function_and_tangent...
sage: plot_function_and_tangent(sec(x), -pi/2,
    xmin=-pi/2, xmax=0)
```



ouch. need to adjust ymax

Examples

```
sage: def plot_function_and_tangent...
sage: good_sec = plot_function_and_tangent(
        sec(x),
        xmin=-pi/2,x0=-pi/4,xmax=0)
sage: show(good_sec,ymax=3)
```



Note: $\sec(x)$ does not work in older versions, apparently because its derivative is not computed

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Scripting?

`script` a sequence of Sage statements saved to a file

- write, save
- load at later date
- no need to type, re-type statements

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Summary

`script` a sequence of Sage statements saved to a file

- write, save
- load at later date
- no need to type, re-type statements

Let's script our `tangent_line()` procedure.

Creating a script

- Back to 'Files'
- Type `calc_utils.sage` in "Filename" box
- Choose "Sage Code (.sage)" in "Create" drop-down
- put `tangent_line()` code in new file
- Save, close
 - `calc_utils.sage` appears in list of files!

Using a script

- Create new Sage worksheet
 - or open old one

- “Attach” the script:

```
sage: attach calc_utils.sage
```

- Try it:

```
sage: tangent_line(x^2, 1)
```

```
2*x - 1
```

Using a script

- Create new Sage worksheet
 - or open old one
- “Attach” the script:
`sage: attach calc_utils.sage`
- Try it:
`sage: tangent_line(x^2, 1)`
`2*x - 1`
- You have now “extended” Sage!
- Can use `tangent_line()` in any worksheet, or even in other scripts, using `%attach` directive

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Summary

- procedures collect several Sage statements into one
 - organizes solutions to problems
 - abstraction makes problem-solving easier
- define using `def (...)` :
- procedures receive *arguments* as data
 - can specify default values
 - procedure does not change arguments, *but...*
 - elements of collections can be changed
- Return value(s) using `return`
- Scripts save procedures for later usage