MAT 305: Mathematical Computing

Interactive worksheets in Sage

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Outline

1. Interactive worksheets
2. Interactive objects
3. Detailed example
4. Summary

You should be in worksheet mode to repeat the examples.

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Interactive worksheets?

An *interactive worksheet* allows a user to visualize and manipulate concepts in a hands-on fashion.

- buttons, sliders, checkboxes
- graphics updated immediately or on demand

Creating interactive worksheets

“Function decorator”: `@interact`

- Place immediately before definition of function
- Formal argument list consists of interact objects
    - input box
    - slider
    - checkbox
    - dropdown menu
    - buttons
    - color selector

Example

```
sage:  @interact
       def i_deriv(f=input_box(label='$f$')):
         if (f != None):
           print 'The derivative of ', f,
                 'is', diff(f)
```

Example

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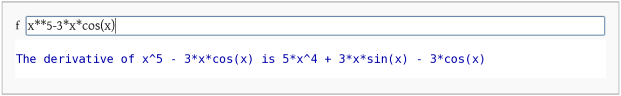


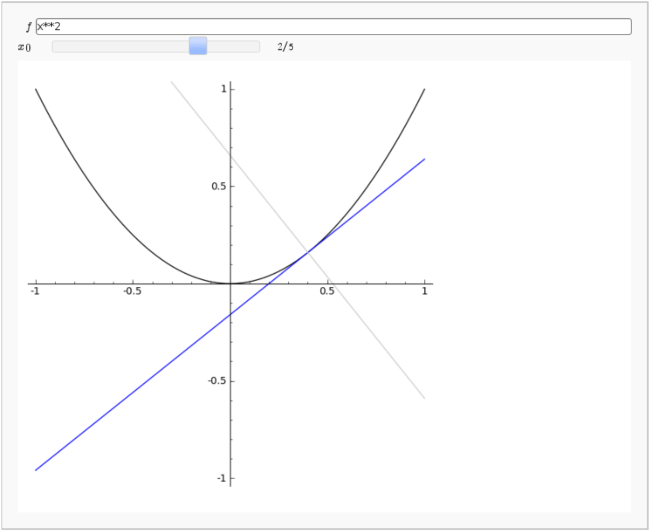
f x**5-3*x*cos(x)

```
The derivative of x^5 - 3*x*cos(x) is 5*x^4 + 3*x*sin(x) - 3*cos(x)
```

Something more visual

```
sage:  xmin, xmax = -1, 1
sage:  @interact
       def i_tan_norm(f=input_box(label=’$f$’),
           x0=slider(xmin,xmax,label=’$x_0$’,
           step_size=1/10,default=0)):
         if f != None and f != ’’:
           y0 = f(x=x0)
           mtan = (diff(f))(x=x0)
           mnorm = -1/mtan
           fplot = plot(f,xmin,xmax,color=’black’)
           tan_plot = plot(mtan*(x-x0)+y0,xmin,xmax)
           norm_plot = plot(mnorm*(x-x0)+y0,xmin,
                            xmax,color=(0.8,0.8,0.8))
           show(fplot+tan_plot+norm_plot,ymin=-1,
                ymax=1,aspect_ratio=1)
```

…the result



f x**2

$x0$ 2/5

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Usage

- argument to interactive function
- *id* = *object*(*options*) where
  - *id* is an argument for the value of the object
  - *object* is one of the object commands given below
  - *options* specify object's properties
    - two options common to all objects

Command options for all objects

- `label` = *label*
  a string that labels the object
  - limited LaTeX
    (`latex()` command can be useful!)
  - compare `label=’x_0’`, `label=’$x_0$’`
- `default` = *value*
  the default value of the object, if any

The `input_box()` command

`input_box(`*options*`)` where *options* include

- `width`: width of box (# letters)

User enters text (function, number, etc.)

Example

```
f = input_box(label='$f$',default=x*cos(x),width=10)
```

The `slider()` command

`slider(`*options*`)` where *options* include

- continuous slider?
  - `start`: minimum value of slider
  - `stop`: maximum value of slider
- discrete slider? two ways
  1. list of values: `start`, no `stop`
  2. range and step size: `start`, `stop`, `step`

User slides knob across line to select value

Example

```
x0 = slider(label='$x_0$',vmin=-1,vmax=1,
            default=0,step_size=1/10)
```

The `checkbox()` command

`checkbox(`*options*`)`

- User sets boolean (on/off or `True`/`False`) value

Example

```
show_tangent = checkbox(label='show tangent',
                         default=True)
```

Choosers

`selector(`*options*`)` where *options* include

- `values`: list of values or (*value*,*label*) pairs
- `buttons`: draw buttons, not a drop-down menu, if `True`
- `nrows`, `ncols`: number of rows or columns of buttons
- `width`: set all buttons to same length (in characters)

User chooses one of several options

Example

```
function = selector(values=[’normal line’,
                          ’tangent line’,
                          ’both’,’neither’])
```

Color selector

`Color(`*color definition*`)` where

- *color definition* is
  - a recognized name for a color
  - an rgb triplet
  - a hex string (don't worry about this one unless you already know what I mean)
- "common" options do not work with this object

User manipulates color using string, circle, box

Example

```
col = Color(0,0,1)
```

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Example problem

Problem

Given f and c, plot f, its derivative df, and a line tangent to f at $x = c$.

Reuse old code

We already know how to compute line tangent to a function: (don’t retype)

```
sage:  def tangent_line(f, a, x=x):
         # redefine f
         f(x) = x
         # point-slope form of a line
         b = f(x=a)
         df(x) = diff(f,x)
         m = df(a)
         result = m*(x - a) + b
         return result
```

Reuse old code

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         # redefine f
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         m = df(a)
         result = m*(x - a) + b
         return result
```

Saved in `calc_utils.py` , so:

```
sage:  %attach calc_utils.sage
```

Interactive function

```
inputs
    $f$, a function
    $c \in \mathbb{R}$
do
    plot $f$
    plot $f'$
    plot line tangent to $f$ at $x = c$
```

Interactive function

```
inputs
    $f$, a function
    $c \in \mathbb{R}$
do
    plot $f$
    plot $f'$
    plot line tangent to $f$ at $x = c$
```

…over what interval? **refine pseudocode**

Interactive function

```
inputs
    $f$, a function
    $a, b \in \mathbb{R}$
    $t \in [0, 1]$
do
    plot $f$ on $[a, b]$
    plot $f'$ on $[a, b]$
    let $c = t(b - a)$
    plot on $[a, b]$ line tangent to $f$ at $x = c$
```

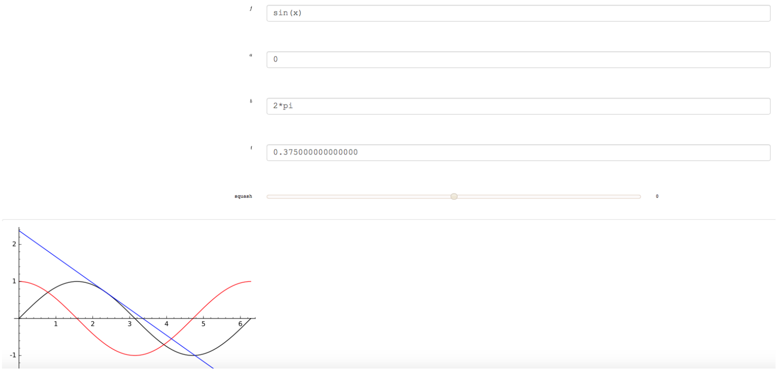
Interactive function

```
inputs
    $f$, a function
    $a, b \in \mathbb{R}$
    $t \in [0, 1]$
do
    plot $f$ on $[a, b]$
    plot $f'$ on $[a, b]$
    let $c = t(b - a)$
    plot on $[a, b]$ line tangent to $f$ at $x = c$
(parametrized $c$ on $[a, b]$ using $t$)
```

Sage code

```
sage: @interact
      def plot_f_df_tline(
            f=input_box(default=sin(x), label=’$f$’),
            a=input_box(default=0, label=’$a$’),
            b=input_box(default=2*pi, label=’$b$’),
            t=slider(0, 1, default=0.375, label=’$t$’),
            w=slider(-19, 19, 1, default=0, label=’squash’)):
          p = plot(f, a, b, color=’black’)
          df = diff(f)
          p = p + plot(df, a, b, color=’red’)
          # t tells us how far to move along [a,b]
          # 0 means a; 1 means b; other values proportional
          c = a + t*(b - a)
          p = p + plot(tangent_line(f, c), a, b)
          show(p, aspect_ratio=(20-w)/20)
```

Result



f sin(x)

a 0

b 2*pi

t 0.375000000000000

squash 0

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Summary

- Interactive worksheets help user visualize, manipulate concepts
- Use `@interact` function decorator
- Several easy-to-define interface objects
- Break functions into parts
  - easy to read
  - easy to reuse
  - easy to change