MAT 305: Lab #9

April 6, 2016

Directions: The usual counsels apply.

1. Create a new worksheet. Set the title to, "Lab #9". Add other information to identify you, as necessary.

2. Select a problem according to the following schema.

If your ID ends with...	...use this function...	...over this interval.
0,1,2	$f(x) = \sin x$	$\left[-\frac{\pi}{3}, \frac{2\pi}{3}\right]$
3,4,5	$f(x) = \cos x$	$\left[-\frac{\pi}{3}, \frac{2\pi}{3}\right]$
6,7,8	$f(x) = \tan x$	$\left[-\frac{\pi}{6}, \frac{\pi}{3}\right]$
other	weird: see me	

Part 1: Derivatives

3. Find the equation of the line tangent to f at $x = \pi/4$. Any computation that can be done with Sage should be evident in your worksheet!

4. Combine the plots of both f and the line tangent to it over the interval given. The curve for f should be black, and have a width of 2. The line should be blue, and have a width of 2.

5. Create an animation with at least 8 frames that shows the approach of the secant line to the tangent line as $x \to \pi/4$ from the left. Reuse the plots of f and the tangent line from above. The secant lines should be red, and have a width of 1. You are free to choose any points you like for the secant, just so long as $x \to \pi/4$ from the left. When you are done, your animation should resemble the one on the course syllabus: for instance, the secant line should proceed back and forth, not just in one direction.

Part 2: Exact integrals

6. Compute the area between f and $g(x) = 1 - x^2$ over the interval given.

7. Combine the plots of both f and g over the interval given. Fill in the area between f and g. The curves for both f and g should be black, with a width of 2. The filling can be any color you like, but make it half-transparent. Add a text label inside the filling which contains the area. Use $\LaTeX$ so that the text label looks nice.

Part 3: Approximate integrals

8. Go back to your Calculus text and review the calculation of arclength with integrals. Write the formula, and in a text cell explain briefly what tool from high school geometry is used to derive the formula.

9. Use Sage to ***approximate*** the arclength of the ellipse $x^2/4+y^2/9=1$. Limit the approximation to 5 sample points, and round your answer to 5 decimal places.

10. Repeat problem 9, this time limiting the approximation to 10 sample points. What part of the answer indicates that you have a more accurate answer?

Part 4: BONUS! (for those with exceptional time and/or motivation)

Animate approximations of the arclength, where each frame shows

- no axes;
- the ellipse, in black, with the curve's width 2;
- six frames of 5 dashed line segments, then 6 dashed line segments, ..., and finally 10 dashed line segments, in red, of width 1;
- a text label in each frame with the corresponding approximation to the arclength, at the center of the ellipse, in black.

This bonus is worth as much as *the entire assignment*. If you wish, you may do Part 4 instead of Parts 1–3. Be sure you know what you are doing; this can take a while.