MAT 305: Lab #7

March 30, 2016

In class we studied the **Fibonacci sequence**

$$f_1 = 1, \quad f_2 = 1, \quad f_{n+2} = f_n + f_{n+1}.$$

The Fibonacci numbers are one example of what mathematicians now call a **Lucas sequence**. (More information at the link.) We usually define Lucas sequences recursively, but you can find a "closed formula" in a manner similar to what we did in class for the Fibonacci sequence.

Let a, b, c, and d be the first two numbers of your student ID. The sequence

$$\ell_1 = a, \quad \ell_2 = b, \quad \ell_{n+2} = c\ell_n + d\ell_{n+1}$$

is a Lucas sequence that we'll call the "*[insert your last name here]* sequence."

1. In a Sage text cell, state the definition of the *[insert your last name here]* sequence. Use LaTeX!
2. In the same cell, list the first five numbers of the *[insert your name here]* sequence.
3. Define a matrix L and a vector $\mathbf{v}$ which generate the sequence. For instance, if the first four digits of your ID are 1, 2, 8, and 9 then

$$L = \begin{pmatrix} 8 & 9 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad \mathbf{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

   Compute $L\mathbf{v}$, $L^2\mathbf{v}$, $L^3\mathbf{v}$, $L^4\mathbf{v}$, and $L^5\mathbf{v}$ in Sage, and compare the results to #2. If they differ, either in #2 or #3 is wrong. *Or* I have a typo. Ask me, and/or fix it before continuing.

4. Compute L's "eigendata." Extract the eigenvectors and eigenvalues and have Sage convert them to *radical* form. (Numbers should no longer end in question marks.)
5. Construct matrices Q and Λ such that $L = Q\Lambda Q^{-1}$. Use Sage to verify that $L = Q\Lambda Q^{-1}$.
6. Construct the matrix $M = \left(Q\Lambda Q^{-1}\right)^n$.
   Hint: The lecture notes discuss this; it requires some knowledge of linear algebra.
7. Use the product of M and $\mathbf{v}$ to find the closed form of the *[insert your name here]* sequence.
   Hint: Again, the notes should come in handy here if you need help.
8. Use the closed form to compute the first five numbers of the *[insert your name here]* sequence, and compare your results to what you found in #2 and #3. If they differ, you have a problem or I have a typo; ask me and/or fix it!