

MAT 305: Lab #6

March 11, 2016

Remark. There are two problems; one on the front, one on the back. As usual, try to organize the computational cells in your worksheet under text cells that separate problems, parts of problems, add commentary, etc.

1. Define two symbolic variables a and b . Let

$$A = \begin{pmatrix} \cos a & \sin a \\ -\sin a & \cos a \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} \cos b & \sin b \\ -\sin b & \cos b \end{pmatrix}.$$

- Compute AB in Sage. Use your knowledge of trigonometry to specify a simpler form than what Sage gives. Use L^AT_EX to write this simpler form in a text box below the computation of AB .
- Extract, and give a name to, the entry in the first row and column of AB .

$$\begin{pmatrix} \text{this one} & \text{not this one} \\ \text{nor this one} & \text{certainly not this one} \end{pmatrix}$$

Try to give it a meaningful name, not just x , which is a bad idea, anyway.

- In a new computational cell, type the name you just created for that entry. Then, type a dot (period). Then press tab. Look through the names that pop up for a command that might reduce the trigonometric expression to something simpler. Use that command to confirm your result in part (b).
- Define a new matrix, C , obtained by substituting the value $a = \pi/3$ into A .
- Let \mathbf{v} be the vector defined by $(5, 3)$.
- Compute the vectors $C\mathbf{v}$, $C^2\mathbf{v}$, $C^3\mathbf{v}$, $C^4\mathbf{v}$, $C^5\mathbf{v}$.
- Plot the vectors \mathbf{v} , $C\mathbf{v}$, $C^2\mathbf{v}$, $C^3\mathbf{v}$, $C^4\mathbf{v}$, $C^5\mathbf{v}$ in different colors (your choice). Plot them as arrows or points, but not as step functions. Please combine them into a single plot, rather than making six different plots.
- What is the geometric effect of multiplying \mathbf{v} by the matrix C repeatedly? What do you predict $C^{60}\mathbf{v}$ would look like? What about $C^{1042}\mathbf{v}$?

2. Let M be the 2×2 matrix defined by the first four numbers in your student ID.
- Use Sage to find the eigenvalues and eigenvectors of M . There should be two distinct eigenvectors, but it's possible you'll get only one, with a multiplicity of 2. In that case, modify the entries of your matrix *very slightly* so that it produces two distinct eigenvectors.
 - Extract the eigenvectors and name them v_1 and v_2 .
Hint: For full credit, *extract* them using the bracket operator; do not define new vectors.
 - Let K be the list of vectors obtained from the 10 products $M^i v_1$ for $i = 1, 2, \dots, 10$. Use a **for** loop to define this list; you will lose the vast majority of points on this part if you do them one at a time.
Hint: Be careful. The expressions `range(10)` and `xrange(10)` commands start at 0; you want to start at 1.
 - Let L be the list of vectors obtained from the 10 products $M^i v_2$ for $i = 1, 2, \dots, 10$. Use a **for** loop to define this list; you will lose the vast majority of points on this part if you do them one at a time.
 - Define `p=Graphics()` and `q=Graphics()`. (This defines an "empty" plot.)
 - Use a **for** loop to add a plot of each vector in K to p . The first vector should be red; successive vectors should shift increasingly towards blue, until the last one is completely blue. Here's how I'd suggest shifting from red to *green*:

```
for i in xrange(len(L)):
    p += plot(L[i], color=((10-i)/10, i/10, 0), zorder=-i)
```
 - Use a **for** loop to add a plot of each vector in L to q . Vectors should shift from purple (red and blue) to yellow (red and yellow).
 - Show p in one cell; show q in another.
 - Describe how the result is consistent with what we said in class about eigenvalues and eigenvectors. If it isn't clear what I mean, try adjusting the `xmin`, `xmax`, `ymin`, `ymax` values to see a smaller portion of the graph.