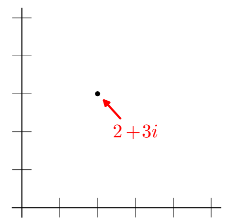
# Lab #4

MAT 305

Spring 2016

*Remark.* In this assignment, we view all solutions as complex numbers $a+bi$, where $i^2=-1$. When you are asked to "plot $a+bi$ on the complex plane," plot it as the point $(a,b)$. So, for instance, a plot of the complex number $2+3i$ would give you the point $(2,3)$:




1. Create a new worksheet. Set the title to, "Review #3". Add other information to identify you, as necessary.
2. Use Sage to solve the equation $x^2-1=0$. Plot all solutions on one complex plane. It's not very interesting, is it?
3. Use Sage to solve the equation $x^3-1=0$. Plot all solutions on one complex plane, not the same as before. This is a little more interesting.
4. Use Sage to solve the equation $x^4-1=0$. Plot all solutions on a third complex plane. This might be a little boring, again.
5. Use Sage to solve the equation $x^5-1=0$. Plot all solutions on a fourth complex plane. (You will probably need to use `real_part()` and `imag_part()` here.) This figure should be arresting, especially if you connect the dots, but don't do that; just plot the points, note the result, and move on.
6. Can you guess? Yep, use Sage to solve the equation $x^6-1=0$. Plot all solutions on yet another complex plane. This is probably getting boring, if only because you can probably guess the result not only of this one, but also of…
7. Use Sage to solve the equation $x^7-1=0$. Plot all solutions on another complex plane. If you understand the geometric pattern, move on; if not, complain. Or, if you want to save time, you can probably guess that I'll tell you to plot the solutions for a few more examples of $x^n-1=0$, only with larger values of $n$. Sooner or later, you should perceive a geometric pattern.

8. In a text cell, explain why the geometric pattern of the images justifies the assertion that the solutions to $x^n - 1$ all have the form
$$\cos\frac{2\pi k}{n} + i\sin\frac{2\pi k}{n}.$$
Be sure to explain what on earth $k$ and $n$ have to do with it. If it helps, look first at the parametric plot of $\cos(2\pi t) + i\sin(2\pi t)$ for $t \in [0,1]$, remembering as before that you plot $a + bi$ as $(a, b)$.

   Before you raise your hand in panic, take a moment, then a deep breath, and think back to the meaning of sine and cosine. Look in a textbook if you have to. You can do this, honest!

   Likewise, don't ask me how to create a parametric plot. *We went over it,* so if you don't remember, look it up in the notes.

9. Define $\omega = \cos\frac{2\pi}{6} + i\sin\frac{2\pi}{6}$. Using a **for** loop in Sage, compute $\omega$, $\omega^2$, ..., $\omega^6$. Plot each number on the complex plane, and describe how the result is consistent with your answers to #6 and #8.

10. Now suppose $\omega = \cos\frac{2\pi}{n} + i\sin\frac{2\pi}{n}$. *By hand,* work out an explanation why
$$\omega^k = \cos\frac{2\pi k}{n} + i\sin\frac{2\pi k}{n}. \tag{1}$$
Type this explanation in your Sage worksheet, using LaTeX to make it all purty-like. If you know proof by induction, that will work. Otherwise, take the following approach:

    - Explain why equation (1) is true for $k = 1$.
    - Expand $\omega^2$ and use some trigonometric identities you're supposed to remember to simplify to the value of equation (1) for $k = 2$.
    - Do the same for $\omega^3$.
    - Finally, point to a pattern in the previous two steps that you can repeat *ad infinitum*, so that if equation (1) is true for *some* value of $k$, it's also true for the *next* value of $k$.

11. Time to up our game.[1] Experiment with $x^n - a$ for some nice values of $a$ and a sufficiently large value of $n$. What do you see? Formulate a conjecture as to the geometric form of these solutions.

    (The term "nice values of $a$" it can mean one of two things. *First,* it can mean numbers of the form $a = b^n$; for instance, if $n = 6$, then you could let $b = 3$, and you would have $a = 3^6 = 729$. You'd then work with the equation $x^6 - 729$, which is not as scary as it looks. Honest! *Second,* you could take the Bob Ross approach, in which case any number is a nice value because, really, all numbers are nice, happy numbers. That's harder, though.)

12. As before, make it all look nice, with sectioning, commentary in text boxes, at least a *little* LaTeX, etc.

[1] I'd rather say, "Let's generalize," as "up our game" just don't roll off the tongue in the same pleasant way, but They tell me that "up our game" is more current. Don't ask who They are, and don't look at me like I'm crazy. Of course I am.