MAT 305: Review #2

February 10, 2016

1. Create a new worksheet. Set the title to, "Review #2". Add other information to identify you, as necessary.

Part 1: Implicit plots

2. Select a problem according to the following schema.

If your ID ends with...	...use this equation.
1,2	$x^3 + x = y^2$
4,5	$\left(x^2 + y^2\right)^2 = \frac{25}{4} x y^2$
6,7	$x^4 - x^2 y + y^4 = 1$
other	weird: see me

3. First, create and display an implicit plot of the equation on the region $[-2.25, 2.25] \times [-2.25, 2.25]$, using at least 300 points. The curve can be any color you like, as long as it's black.[1]

4. Choose an x-value in the interval $[-2, 2]$. Using your math skills (not Sage's, nor anyone else's — getting help from me is okay) find the equation of a line tangent to the curve at that point. Write the equation of this line in a text cell. Use basic LaTeX commands to make sure it looks nice. *Note:* There may be more than one y-value for that x-value, so there may be more than one line tangent to the curve for that value of x, but you need plot only one. Don't pick a point where the tangent line is horizontal or vertical; that would be cheating.

5. Verify your claim by combining the plot of the curve with a plot of the tangent line. The line should be red. Put a red dot at the point where the line and curve intersect. Make the dot large enough to stand out; I don't think the default dot size stands out.
   Note: I have updated the lecture on plotting, to include the option that controls how the plot objects are "stacked above each other." Use the relevant option to ensure the point is on top, and the tangent line on bottom.

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[1] That's a joke, but make it black, anyway. No apologies to Henry Ford.

Part 2: Parametric plots

6. Plot a black Bézier curve. Remember that this involves four control points (x_0, y_0), (x_1, y_1), (x_2, y_2), (x_3, y_3) and the equations

$$\begin{cases} x(t) & = x_0(1-t)^3 + 3x_1 t(1-t)^2 + 3x_2 t^2(1-t) + x_3 t^3 \\ y(t) & = y_0(1-t)^3 + 3y_1 t(1-t)^2 + 3y_2 t^2(1-t) + y_3 t^3 \end{cases}$$

where $t \in [0, 1]$. You can choose the four control points at random, but don't use the ones I used in class. Feel free to make a loop, if you can. Add red dashed lines that connect the 1st and 2nd control points (x_0, y_0) and (x_1, y_1), and the 3rd and 4th control points (x_2, y_2) and (x_3, y_3), allowing the viewer to see how the control points relate to tangent lines.

7. Use your graph to write a short description of the relationship between the control points and the lines.

Part 3: Animate!

8. Animate the Bézier curve:

   - Change the values of the 2nd and 3rd points 12 times. (Don't change them by very much, only a little, so the curve doesn't jump around too abruptly.)
   - For each change, create a new graph. Each graph should be identical to the one you created in step 6, except that the 2nd and 3rd control points are different.
   - Animate all the graphs using the `animate()` command, and `show()` it.

Part 4: Make it look good!

9. Add text cells throughout the document that clearly delineate the parts. It should be clear where each part begins and ends. In Part 1, a text cell should state specifically what value of x you chose; in Part 2, a text cell should state specifically what control points you used; in Part 3, a text cell should state clearly the sequence of adjustments made to the 2nd and 3rd control points. You should also add some explanatory text before each plot: "This is the first plot, with points…", "This is the second plot, with points…", … and finally "This is the animation of all the frames." **Use LaTeX for mathematical expressions and statements;** don't forget the reference of useful LaTeX commands on the class website.