# MAT 305: Mathematical Computing

## Interactive worksheets in Sage

John Perry

University of Southern Mississippi

Spring 2016

# Outline



*You should be in worksheet mode to repeat the examples.*

# Outline

# Interactive worksheets?

An *interactive worksheet* allows a user to visualize and manipulate concepts in a hands-on fashion.

- buttons, sliders, checkboxes
- graphics updated immediately or on demand

# Creating interactive worksheets

"Function decorator": `@interact`

- Place immediately before definition of function
- Formal argument list consists of interact objects
  - input box
  - slider
  - checkbox
  - dropdown menu
  - buttons
  - color selector

# Example

```
sage:  @interact
       def i_deriv(f=input_box(label='$f$')):
         if (f != None):
           print 'The derivative of ', f,
                 'is', diff(f)
```

# Example

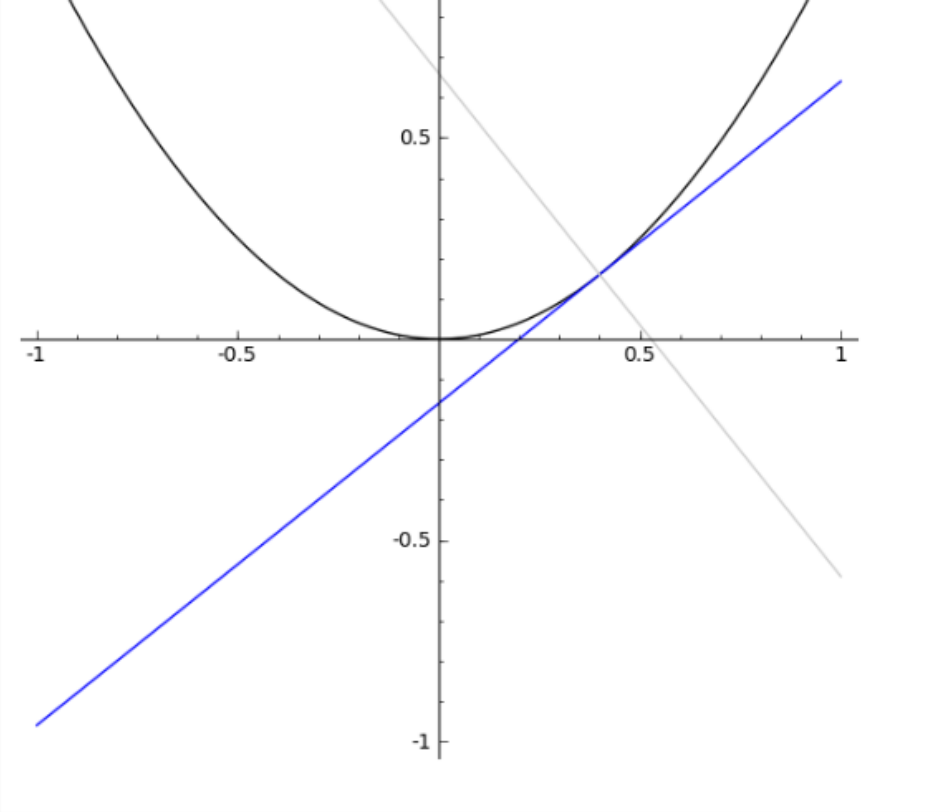
```
sage:  @interact
       def i_deriv(f=input_box(label='$f$')):
         if (f != None):
           print 'The derivative of ', f,
                 'is', diff(f)
```

f x**5-3*x*cos(x)

```
The derivative of x^5 - 3*x*cos(x) is 5*x^4 + 3*x*sin(x) - 3*cos(x)
```

# Something more visual

```
sage:  xmin, xmax = -1, 1
sage:  @interact
       def i_tan_norm(f=input_box(label='$f$'),
           x0=slider(xmin,xmax,label='$x_0$',
           step_size=1/10,default=0)):
         if f != None and f != '':
           y0 = f(x=x0)
           mtan = (diff(f))(x=x0)
           mnorm = -1/mtan
           fplot = plot(f,xmin,xmax,color='black')
           tan_plot = plot(mtan*(x-x0)+y0,xmin,xmax)
           norm_plot = plot(mnorm*(x-x0)+y0,xmin,
                       xmax,color=(0.8,0.8,0.8))
           show(fplot+tan_plot+norm_plot,ymin=-1,
                ymax=1,aspect_ratio=1)
```


0.5
-0.5
-1
-1
-0.5
0.5
1

# Outline

# Usage

- argument to interactive function
- *id* = *object*(*options*) where
  - *id* is an argument for the value of the object
  - *object* is one of the object commands given below
  - *options* specify object's properties
    - two options common to all objects

# Command options for all objects

- `label` = *label*
  a string that labels the object
  - limited LaTeX
    (`latex()` command can be useful!)
  - compare `label='x_0'`, `label='$x_0$'`

- `default` = *value*
  the default value of the object, if any

# The `input_box()` command

`input_box(`*options*`)` where *options* include

- `width`: width of box (# letters)

User enters text (function, number, etc.)

## Example

```
f = input_box(label='$f$',default=x*cos(x),width=10)
```

# The `slider()` command

`slider(`*options*`)` where *options* include

- continuous slider?
  - `start`: minimum value of slider
  - `stop`: maximum value of slider
- discrete slider? two ways
  1. list of values: `start`, no `stop`
  2. range and step size: `start`, `stop`, `step`

User slides knob across line to select value

## Example

```
x0 = slider(label='$x_0$',vmin=-1,vmax=1,
            default=0,step_size=1/10)
```

# The `checkbox()` command

`checkbox(`*options*`)`

- User sets boolean (on/off or `True`/`False`) value

## Example

```
show_tangent = checkbox(label='show tangent',
                        default=True)
```

# Choosers

`selector(`*options*`)` where *options* include

- `values`: list of values or (*value*,*label*) pairs
- `buttons`: draw buttons, not a drop-down menu, if `True`
- `nrows`, `ncols`: number of rows or columns of buttons
- `width`: set all buttons to same length (in characters)

User chooses one of several options

## Example

```
function = selector(values=['normal line',
                            'tangent line',
                            'both','neither'])
```

# Color selector

`Color(`*color definition*`)` where

- *color definition* is
  - a recognized name for a color
  - an rgb triplet
  - a hex string (don't worry about this one unless you already know what I mean)
- "common" options do not work with this object

User manipulates color using string, circle, box

## Example

```
col = Color(0,0,1)
```

# Outline

# Example problem

## Problem

Given $f$, $a$, $b$, and $n$, use $n$ rectangles to approximate $\int_a^b f(x)\,dx$. Use left endpoints to approximate the height of each rectangle.

# Function definition

How can we make this interactive? Let user define:

- $f$, $a$, $b$ as input boxes
- $n$ as slider from 2 to 10
- color of boxes

# Function definition

How can we make this interactive? Let user define:

- $f$, $a$, $b$ as input boxes
- $n$ as slider from 2 to 10
- color of boxes

$\therefore$ function definition:

```
@interact
def
i_left_sums(f=input_box(default=x**2,label='$f$'),
            a=input_box(default=0,label='$a$'),
            b=input_box(default=1,label='$b$'),
            n=slider(start=range(2,11),default=2,
                     label='$n$'),
            boxcolor=Color(0.5,0.5,0.5)):
```

# Avoid complicated functions

Major subtasks $\longrightarrow$ functions:

- `left_Riemann_sum()` to approximate area
- `left_Riemann_rectangles()` to make plots

# Approximating area

- Already solved approximation of $\int_a^b f(x)\,dx$ using left endpoints. ***Reuse old work!***

- Prior to `@interact`, paste old left Riemann sum code.

```
def left_Riemann_sum(f, a, b, n, x=x):
  Delta_x = (b-a)/n
  L = range(n)
  S = 0
  for i in L:
    xi = a + i*Delta_x
    S = S + f({x:xi})*Delta_x
  return S
```

# Graphics

- plotting $f$ is easy
  `fplot = plot(f,a,b)`

# Graphics

- plotting $f$ is easy
  `fplot = plot(f,a,b)`
- plotting rectangles: use `polygon2d()` command
  `polygon2d([`*lower_left*`,` *upper_left*`,`
  *upper_right*`,` *lower_right*`])`
- use **for** loop to combine rectangles into plot

# Graphics

- plotting $f$ is easy
  `fplot = plot(f,a,b)`
- plotting rectangles: use `polygon2d()` command
  `polygon2d([`*lower_left*`,` *upper_left*`,`
  *upper_right*`,` *lower_right*`])`
- use **for** loop to combine rectangles into plot

```
combo = fplot
L = range(n)
for i in L:
  xi = a + i*Delta_x
  yi = f(x)
  combo = combo + polygon2d([(xi,0),(xi,yi),
          (xi+Delta_x,yi),(xi+Delta_x,0)],
          color=boxcolor,alpha=0.75)
```

# Encapsulate as function

Also prior to `@interact`:

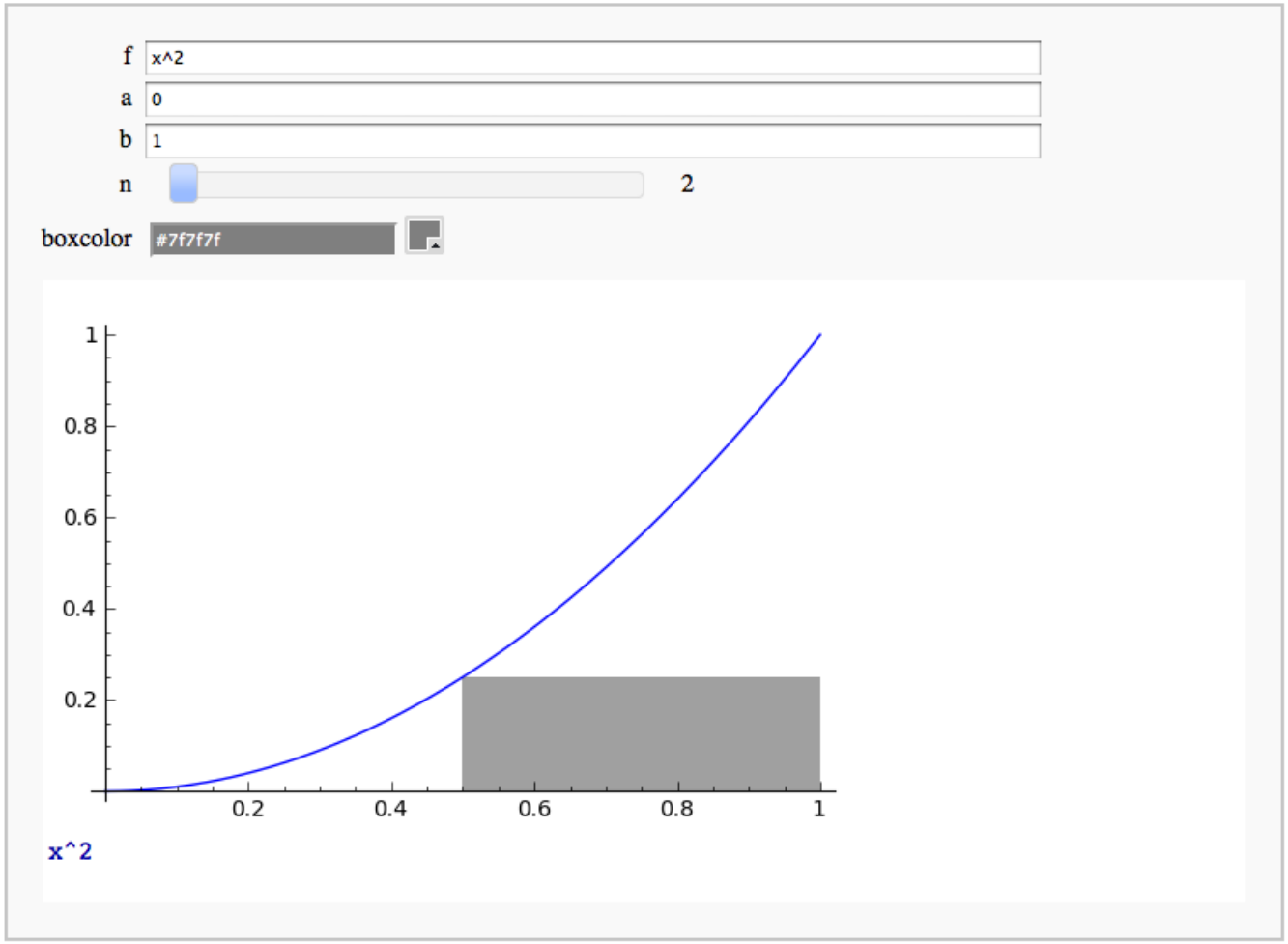
```
def
left_Riemann_rectangles(f,a,b,n,x=x,boxcolor='red'):
  fplot = plot(f,a,b)
  combo = fplot
  Delta_x = (b-a)/n
  L = range(n)
  for i in L:
    xi = a + i*Delta_x
    yi = f({x:xi})
    combo = combo + polygon2d([(xi,0),(xi,yi),
            (xi+Delta_x,yi),(xi+Delta_x,0)],
            color=boxcolor,alpha=0.75)
  return combo
```

# Combine pieces

Call both from `i_left_sums()`:

```
@interact
def i_left_sums(f=input_box(default=x**2),
                ...
                boxcolor=Color(0.5,0.5,0.5)):
  approx = left_Riemann_sum(f,a,b,n)
  riemann_plot = left_Riemann_rectangles(f,a,b,n,
                                         boxcolor)
  show(riemann_plot)
  print approx
```

# The final product



# Outline

# Summary

- Interactive worksheets help user visualize, manipulate concepts
- Use `@interact` function decorator
- Several easy-to-define interface objects
- Break functions into parts
  - easy to read
  - easy to reuse
  - easy to change