MAT 305: Review #6

April 4, 2014

Directions: The usual counsels apply.

Part I
Modifying an already-written function

1. In class, we wrote a function, `method_of_bisection`, that approximates the root of a function to 2 decimal places. Rewrite the function so that it approximates the root of a function to d decimal places, where d is an argument specified by the user.

Part II
Functions with definite loops

On a previous assignment, you had to decide whether a given set (the *quaternions*) was a group. This required a lot of calculations that you had to request by hand. This time, you'll use functions and loops, so that you can check this for arbitrary sets and operations.

2. Write pseudocode for four algorithms, which will determine whether a finite set S and an operation $\otimes$ satisfy:

   - *closure,* that is, $x \otimes y \in S$ for all $x, y \in S$;
   - *associative,* that is, $x \otimes (y \otimes z) = (x \otimes y) \otimes z$ for all $x, y, z \in S$;
   - *identity,* that is, we can find $\iota \in S$ such that $x\iota = x$ and $\iota x = x$ for all $x \in S$;
   - *inverse,* that is, for any $x \in S$, we can find $y \in S$ such that $xy = yx = \iota$.

   Be sure to specify the inputs needed, and remember that pseudocode should look like english with mathematics, not like Python. *You will lose points* for using Python-isms in pseudocode. **I will do an example of one property in class, which should make the others easier.**

3. Implement three of your algorithms as a Sage function. The operation should be used as a function with two inputs x and y; you can use it by invoking `set_operation(x,y)`. **I will do an example of one property in class, which should make the others easier.**

4. Test your answers on each of the following possibilities:

   (a) S is the quaternions, and $\otimes$ is matrix multiplication.

   (b) S is the set of integers from 0 to 11, and $\otimes$ is addition, *modulo 12*.

   (c) S is the set of integers from 0 to 11, and $\otimes$ is multiplication, *modulo 12*.