

# MAT 305: Review #5

March 17, 2014

**Directions:** The usual counsels apply.

1. Create a new worksheet. Set the title to, “Review #5”. Add other information to identify you, as necessary.
2. Select a problem according to the following schema.

If your ID ends with...	... use this function...	... over this interval.
0,1,2	$f(x) = \sin x$	$[-\frac{\pi}{3}, \frac{2\pi}{3}]$
4,5	$f(x) = \cos x$	$[-\frac{\pi}{3}, \frac{2\pi}{3}]$
6,9	$f(x) = \sec x$	$[-\frac{\pi}{6}, \frac{\pi}{3}]$
other	weird: see me	

## Part 1: Derivatives

3. Find the equation of the line tangent to  $f$  at  $x = \pi/4$ . Any computation that can be done with Sage should be evident in your worksheet!
4. Combine the plots of both  $f$  and the line tangent to it over the interval given. The curve for  $f$  should be black, and have a width of 2. The line should be blue, and have a width of 2.
5. Create an animation with at least 8 frames that shows the approach of the secant line to the tangent line as  $x \rightarrow \pi/4$  from the left. Reuse the plots of  $f$  and the tangent line from above. The secant lines should be red, and have a width of 1. You are free to choose any points you like for the secant, just so long as  $x \rightarrow \pi/4$  from the left. When you are done, your animation should resemble the one on the course syllabus: for instance, the secant line should proceed back and forth, not just in one direction.

## Part 2: Exact integrals

6. Compute the area between  $f$  and  $g(x) = 1 - x^2$  over the interval given.
7. Combine the plots of both  $f$  and  $g$  over the interval given. Fill in the area between  $f$  and  $g$ . The curves for both  $f$  and  $g$  should be black, with a width of 2. The filling can be any color you like, but make it half-transparent. Add a text label inside the filling which contains the area.

### Part 3: Approximate integrals

8. Go back to your Calculus text and review the calculation of arclength with integrals. Write the formula, and in a text cell explain briefly what geometric tool is used to approximate it.
9. Use Sage to approximate the arclength of the ellipse  $x^2/4+y^2/9 = 1$ . Limit the approximation to 5 sample points, and round your answer to 5 decimal places.
10. Repeat problem 9, this time limiting the approximation to 10 sample points. What part of the answer indicates that you have a more accurate answer?

### Part 4: BONUS! (for those with exceptional time and/or motivation)

Animate approximations of the arclength, where each frame shows

- no axes;
- the ellipse, in black, of width 2;
- 5, 6, 7, ..., 10 dashed line segments, in red, of width 1;
- a text label with the corresponding approximation to the arclength, at the center of the ellipse, in black.

This bonus is worth as much as *the entire assignment*. If you wish, you may do Part 4 instead of Parts 1–3. Be sure you know what you are doing; this can take a while.