MAT 305: Review #3

February 10, 2014

Remark. In this assignment, we view all solutions as complex numbers $a + bi$, where $i^2 = -1$. When you are asked to "plot $a + bi$ on the complex plane," plot it as the point (a, b).

1. Create a new worksheet. Set the title to, "Review #3". Add other information to identify you, as necessary.

2. Use Sage to solve the equation $x^2 - 1 = 0$. Plot the solutions on the complex plane. It's not very interesting, is it?

3. Use Sage to solve the equation $x^3 - 1 = 0$. Plot the solutions on the complex plane. This is a little more interesting.

4. Use Sage to solve the equation $x^4 - 1 = 0$. Plot the solutions on the complex plane. This might be a little boring, again.

5. Use Sage to solve the equation $x^5 - 1 = 0$. Plot the solutions on the complex plane. This figure should be arresting, especially if you connect the dots, but don't do that; just plot the points, note the result, and move on.

6. Can you guess? Yep, use Sage to solve the equation $x^6 - 1 = 0$. Plot the solutions on the complex plane. This is probably getting boring, if only because you can probably guess the result not only of this one, but also of...

7. Use Sage to solve the equation $x^7 - 1 = 0$. Plot the solutions on the complex plane. If you understand the geometric pattern, move on; if not, complain. Or, if you want to save time, you can probably guess that I'll tell you to plot the solutions for a few more examples of $x^n - 1 = 0$, only with larger values of n. Sooner or later, you'll perceive a geometric pattern.

8. In a text cell, explain why the geometric pattern of the images justify the assertion that the solutions to $x^n - 1$ all have the form

   $$\cos\frac{2\pi k}{n} + i\sin\frac{2\pi k}{n}.$$

   Be sure to explain what on earth k and n have to do with it. If it helps, first look at the parametric plot of $\cos(2\pi t) + i\sin(2\pi t)$ for $t \in [0, 1]$, remembering as before that you plot $a + bi$ as (a, b).

9. Time to up our game.[1] Experiment with $x^n - a$ for some nice values of a and some sufficiently large values of n. What do you see? Formulate a conjecture as to the form of these solutions.

10. As before, make it all look nice, with sectioning, commentary in text boxes, at least a *little* LaTeX, etc. I'll be happy to help with LaTeX if you need it.

[1] I'd rather say, "Let's generalize," as such cliches don't fit my personality one whit, but They tell me it reaches the students better. (Don't ask who They are.)