

MAT 305: Mathematical Computing

Introduction to Mathematical Computing

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Outline

- ① What this class is about
- ② Computer programming
- ③ Summary

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- 3 Summary

Description

- **Online:** Introduction to a computer algebra system using calculus-based projects. Students will solve mathematical problems in the **MAPLE** environment which require an understanding of calculus concepts.
- **Syllabus:** Introduction to a computer algebra system using calculus-based projects. Students will solve mathematical problems in the **Sage** environment which require an understanding of calculus concepts.

Problem solving or programming?

- This class is about mathematics
 - Problem solving
 - (new!) Introduce ideas of higher mathematics: matrices, modular arithmetic
- Some problems best attacked with a computer
 - Long
 - Experimentation
 - Repetitive/tedious
- Computers require instructions, called **programs**
- We study *some* programming, but class not about programming

Sage?

- Software for Algebra and Geometry Exploration
- Computer Algebra System “started” by William Stein



- Access to other CASs
 - Calculus: Maxima, SymPy, ...
 - Linear Algebra: M4RI, Linbox, PARI, ...
 - Commutative Algebra: SINGULAR, Macaulay, ...
 - Group theory: GAP, ...
 - etc.

Why Sage?

- “Free” software
 - “Free as in beer”: no cost to you
 - Downloading free
 - Installing free
 - Copying free
 - Bug fixes free
 - Future versions free
 - “Free as in speech”:
 - Open-source software
 - No secret algorithms
 - Can study implementation
 - Can correct, improve, contribute

Free mathematics

Theorem

There are infinitely many primes.

Proof.

- Consider finite list of primes, q_1, q_2, \dots, q_n .
- Let $p = q_1 q_2 \cdots q_n + 1$.
- **Fact:** since $p \neq 1$, divisible by at least one prime
- By Division Theorem, p not divisible by any q_i (remainder 1, not 0).
- p divisible by unlisted prime q_{n+1} !
- \therefore no finite list, lists all primes.



Secret mathematics

Theorem

There are infinitely many primes.

Proof.

“I have discovered a truly marvelous proof of this, which this margin is too narrow to contain.”[†] □

[†]Real quote, different theorem.

Analogy

Proprietary mathematics

Theorem

There are infinitely many primes.

Proof.

Trade Secret.



But I prefer Maple!

- Fine, buy your own copy
 - Student discount available
 - I will tell you the Maple equivalents for everything we do in Sage
 - You can submit homework as Maple worksheet
- Be warned:
 - Future versions not free
 - Bug fixes not free
 - I used to use Maple and switched to Sage
 - Recent versions disappointed me
 - After you graduate, pay full price
 - Not always backwards compatible (neither is Sage, but Sage is free)

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Why program?

- Programming bridges gap between humans, computers
 - Computers don't understand human languages
 - Humans intuitive, poetic; computers literal, mechanical
 - Computers only understand **on** or **off**
 - (Most) humans don't understand a computer's native language
 - Mathematics literal and precise, but (most) humans don't understand it, either!
 - Even the humans that do, prefer not to talk to the computer in that language
- Control over computer
- Deeper understanding of computer technology

Kinds of computer languages

What this class
is about

Computer
programming

Summary

- Compiled
 - C/C++
 - FORTRAN
 - Go

- Interpreted or scripting
 - BASIC
 - Python
 - Perl

- Mixed (“bytecode”)
 - C# (.NET)
 - Java

Paradigms of computer languages

What this class
is about

Computer
programming

Summary

- Imperative: BASIC, FORTRAN
- Modular: Modula-2
- Object-oriented: Smalltalk
- Functional: Haskell, ML
- Modern (C++, Python) languages blend

Python

- “Sage” primarily Python
- Python also interface between Sage and user
- Not all *components* of Sage in Python:
 - Maxima: LISP
 - Singular: C/C++
- Python also interface between Sage and user

Advantages of Python

- Modern
 - Facilities for object-oriented, functional programming
- Wide distribution, usage
 - Many employers use it
- Flexible
 - Many good packages enhance it
- Compile for efficiency: Pyrex or Cython

Python \neq Sage

- Some Python commands don't work in worksheet mode
 - `input()`
- Sage commands do not work in plain Python

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Summary

- This class about mathematical problem solving
- Uses programming via Sage
- No “secret methods” in Sage: can
 - inspect inner workings
 - modify it, fix it, break it...
- Interface is sound programming language in wide use