INDIVIDUAL ASSIGNMENT 2

MAT 305 SPRING 2013

Due date: 20 Mar 2013

If you view the class syllabus online, you notice an animation at the top of the page. This animation shows a relationship that you learned in Calculus:

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \left(\frac{\Delta y}{\Delta x} \right);$$

that is, the instantaneous rate of change of y, also called its **derivative**, is the limit of the average rates of change of y between x_1 and x_2 as $\Delta x = x_2 - x_1$ approaches zero.

Rely on what you know from Calculus to answer 1–3; answer the rest and submit *all* work as a **Sage worksheet** on https://lydia.st.usm.edu:8080/ .

(1) Describe how the instantaneous rate of change of y appears in the animation.
(2) Describe how the average rates of change of y appear in the animation.
(3) Describe how the relationship between the average rates of change and the instantaneous rate of change appears in the animation.
(4) Use the last digit of your student number to select the following function and point.

if your student number ends with...	let $f(x) = \ldots$	and $a = \ldots$
0,1	$e^{-x}\cos x$	$-\frac{\pi}{4}$
2,3	$e^{-x}\sin x$	$\frac{\pi}{4}$
4,5	$\ln(1+x^2)$	0
6,7	$\frac{1}{1+x^2}$	0
8,9	$\frac{2x}{1+x^2}$	1

(5) Plot f in black over a small neighborhood of $x = a$.
(6) Show that the derivative of f at $x = a$ is 0.
Note: Use Sage to do this; don't do it by hand.
(7) Plot both f and the line tangent to f at $x = a$. Make the tangent line blue.
(8) Choose four x values b_1, b_2, b_3, and b_4 close to $x = a$. Compute the slopes of the secant lines between a and b_i for $i = 1, 2, 3, 4$.
(9) Create four plots, each of which combines f with a blue secant line.

(10) Combine all the plots to obtain an animation similar to the one on my webpage.
Hint: Notice that on the webpage the secant lines move forwards and backwards; your animation should replicate this behavior.