

MAT 305: Mathematical Computing

2-D Graphing

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Fall 2011

Outline

- 1 Basic 2-D objects
- 2 Plotting functions
- 3 Options for displaying plots
- 4 Implicit plots
- 5 Parametric and polar plots
- 6 Saving images and animations
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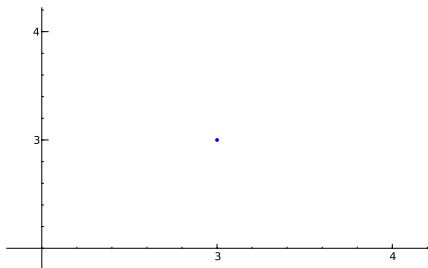
The `point()` command

`point((x_0, y_0), options)` where

- (x_0, y_0) is a Python *tuple*
- *options* include
 - `pointsize`: size of point, default size is 10
 - `rgbcolor`: rgb tuple
 - default color is blue
 - some names allowed: 'red', 'black', etc.
 - more on this later

Example

```
sage: point((3,3))
```



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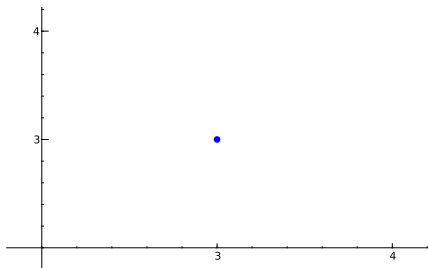
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Example

```
sage: point((3,3),pointsize=30)
```



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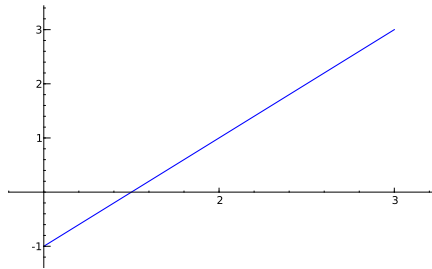
The `line()` command

`line([(x1,y1), (x2,y2), ..., (xn,yn)], options)` where

- (x_1, y_1) and (x_2, y_2) are Python *tuples*
- *options* include
 - **thickness** of curve (default is 1)
 - **linestyle**:
 - `'-'` (solid)
 - `'--'` (dashed)
 - `'-.'` (dash-dot)
 - `':'` (dots)
 - **rgbcolor**

Example

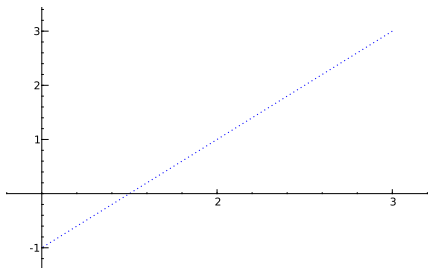
```
sage: line([(3,3),(1,-1)])
```



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```
sage: line([(3,3),(1,-1)],linestyle=':')
```



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The `polygon()` command

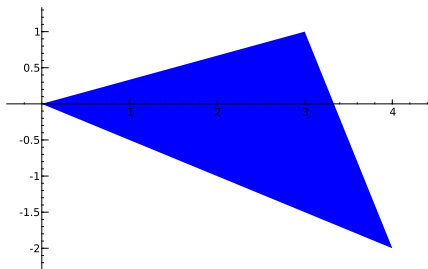
`polygon([(x1,y1), (x2,y2), ..., (xn,yn)], options)` where

- (x_i, y_i) is a Python *tuple* representing a point of the polygon
- *options* include
 - `thickness` of lines (default is 1)
 - `alpha`: transparency of polygon
 - value from 0 to 1
 - 0: invisible; 1 opaque
 - `rgbcolor`

The polygon will be filled. Don't want a filled polygon?
combine lines instead. See below.

Example

```
sage: polygon([(0,0),(3,1),(4,-2)])
```



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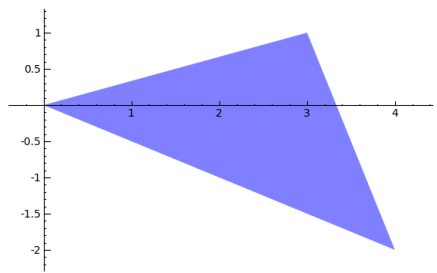
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Example

```
sage: polygon([(0,0),(3,1),(4,-2)],alpha=0.5)
```



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The `text()` command

`text(message, (x0, y0), options)` where

- *message* can be a number, function, or string
 - L^AT_EX strings allowed
- the text is centered over (x_0, y_0)
- *options* include
 - `fontsize` controls text size (default 10)
 - `rgbcolor`

Combine plot objects with +

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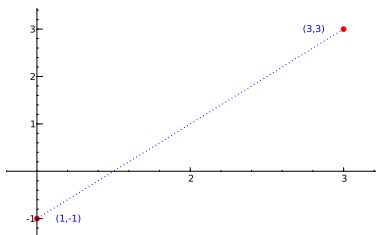
Saving images
and animations

Summary

- Store graphics objects in memory using expressions
- Addition combines simple objects into complex objects

Example

```
sage: point1 = point((3,3),pointsize=30,  
                    rgbcolor='red')  
sage: point2 = point((1,-1),pointsize=30,  
                    rgbcolor='red')  
sage: my_line = line([(3,3),(1,-1)],linestyle=':')  
sage: my_label1 = text('(3,3)',(2.8,3))  
sage: my_label2 = text('(1,-1)',(1.2,-1))  
sage: point1 + point2 + my_line  
      + my_label1 + my_label2
```



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The `plot()` command

`plot($f(x)$, options)` where

- $f(x)$ is an expression
- *options* include
 - `xmin`, `xmax` (*no* `ymin`, `ymax` options in `plot()`)
 - `plot_points`: minimal number of points connected
 - `fill`: to axis, min y value, max y value, a number c , or a function $g(x)$
 - `fillcolor`
 - `rgbcolor`
 - `thickness`
 - `linestyle`

Basic example

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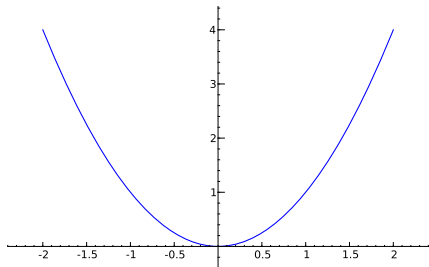
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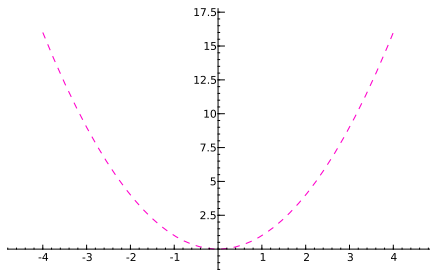
Summary

```
sage: plot(x**2, xmin=-2,xmax=2)
```



Experiment with options!

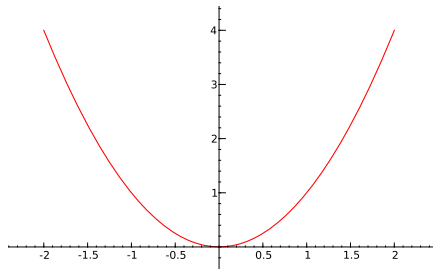
```
sage: plot(x**2, xmin=-4, xmax=4,  
          linestyle='--', rgbcolor=(1,0,0.8))
```



RGB colors

Specify colors using RGB tuples.

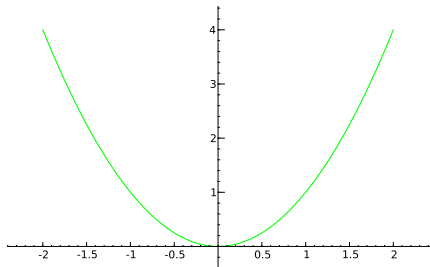
- Red, Green, Blue: primary colors of light
- Pure red: (1,0,0)



RGB colors

Specify colors using RGB tuples.

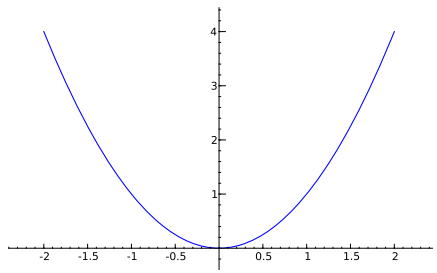
- Red, Green, Blue: primary colors of light
- Pure green: $(0,1,0)$



RGB colors

Specify colors using RGB tuples.

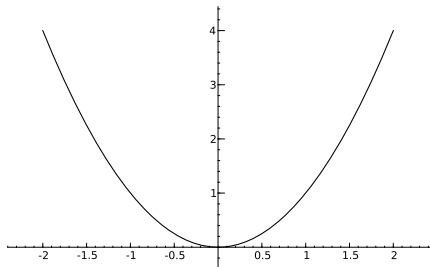
- Red, Green, Blue: primary colors of light
- Pure blue: (0,0,1)



RGB colors

Specify colors using RGB tuples.

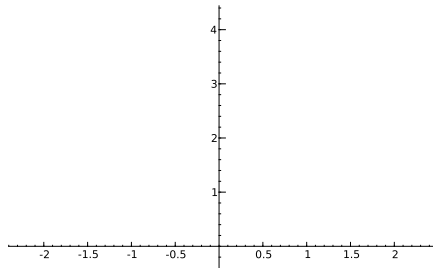
- Red, Green, Blue: primary colors of light
- Black is the absence of color: $(0,0,0)$



RGB colors

Specify colors using RGB tuples.

- Red, Green, Blue: primary colors of light
- White is the presence of all colors: (1,1,1)

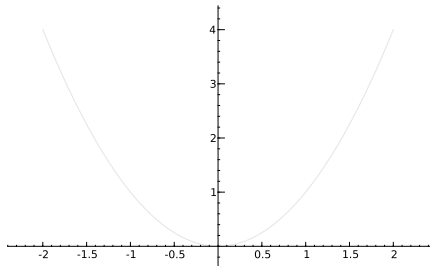


(oops!)

RGB colors

Specify colors using RGB tuples.

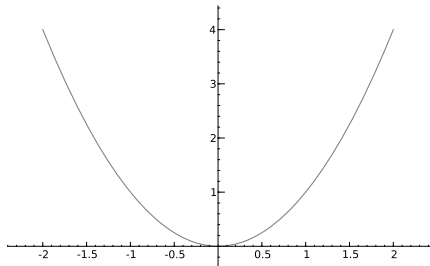
- Red, Green, Blue: primary colors of light
- Gray is an even mixture of the colors: $(0.9,0.9,0.9)$



RGB colors

Specify colors using RGB tuples.

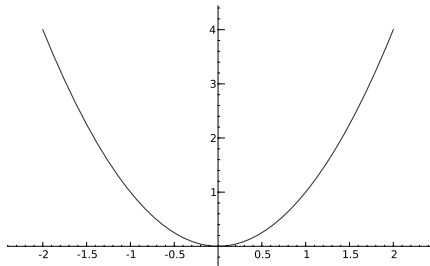
- Red, Green, Blue: primary colors of light
- Gray is an even mixture of the colors: $(0.5, 0.5, 0.5)$



RGB colors

Specify colors using RGB tuples.

- Red, Green, Blue: primary colors of light
- Gray is an even mixture of the colors: $(0.2,0.2,0.2)$



RGB colors

Specify colors using RGB tuples.

- Red, Green, Blue: primary colors of light
- What colors do these tuples represent?
 - $(0.8, 0.6, 0.2)$
 - $(0.9, 0.9, 0)$
 - $(0.3, 0.8, 0.9)$

RGB colors

Specify colors using RGB tuples.

- Red, Green, Blue: primary colors of light
- What colors do these tuples represent?
 - $(0.8, 0.6, 0.2)$
 - $(0.9, 0.9, 0)$
 - $(0.3, 0.8, 0.9)$

brown

yellow

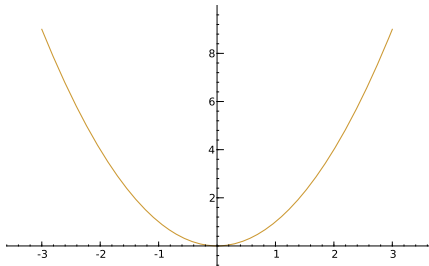
blue-green

Remember colors

- Use expressions to remember colors

```
sage: my_brown = (0.8,0.6,0.2)
```

```
sage: plot(x**2,xmin=-3,xmax=3,color=my_brown)
```



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The show() command

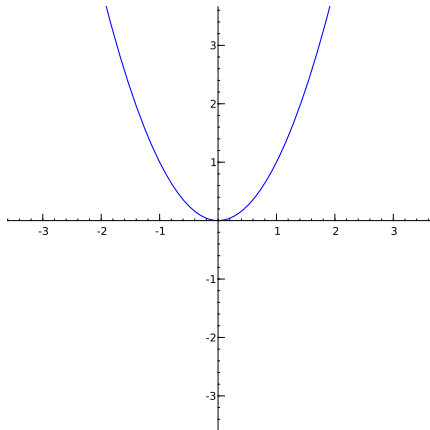
show(*plot object, options*) where

- *plot object* is any object generated by a plot
- *options* include
 - aspect_ratio: width/height
 - 1 makes a “square” graph
 - xmin, xmax
 - ymin, ymax

Example

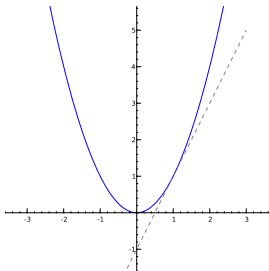
```
sage: myplot = plot(x**2,xmin=-3,xmax=3)
```

```
sage: show(myplot,ymin=-3,ymax=3,aspect_ratio=1)
```



Combine plots

```
sage: par_plot = plot(x**2,xmin=-3,xmax=3)
sage: tan_plot = plot(2*x-1,xmin=-3,xmax=3,
                    color='gray',linestyle='--')
sage: com_plot = par_plot + tan_plot
sage: show(com_plot,ymin=-1,ymax=5,aspect_ratio=1)
```



Different meanings of `xmin`, `xmax`

`plot` x -values for which Sage computes y -values

`show` x -values which Sage shown on the screen

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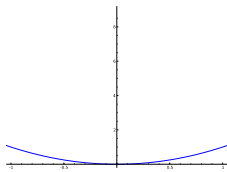
Different meanings of `xmin`, `xmax`

`plot` `x`-values for which Sage computes `y`-values

`show` `x`-values which Sage shown on the screen

```
sage: p = plot(x**2, xmin=-3, xmax=3)
```

```
sage: show(p, xmin=-1, xmax=1)
```



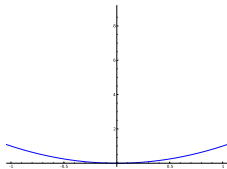
Different meanings of `xmin`, `xmax`

`plot` x-values for which Sage computes y -values

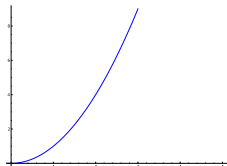
`show` x-values which Sage shown on the screen

```
sage: p = plot(x**2, xmin=-3, xmax=3)
```

```
sage: show(p, xmin=-1, xmax=1)
```



```
sage: show(p, xmin=0, xmax=5)
```



The `animate()` command

`animate((plot1, plot2, ...), options)` where

- `plot1, plot2, ...` are graphics objects
 - each object = one frame
- `options` include
 - `xmin, xmax, ymin, ymax, aspect_ratio`

must show() animation, options:

- `delay` in hundredths of a second (default 20)
- `iterations` (0 = forever, default)
- cannot specify `xmin, xmax, ymin, ymax, aspect_ratio` for animation in `show()`; specify in `animate()` instead

Example

```
sage: par_plot = plot(x**2,xmin=-3,xmax=3,
                    thickness=2,color=(0,0,0))
sage: tan_plot = plot(2*x-1,xmin=-3,xmax=3,thickness=2)
sage: com_plot = par_plot + tan_plot
sage: pink = (1.0,0.5,0.5)
sage: sec1_plot = plot(1,xmin=-3,xmax=3,
                    color=pink,linestyle='--')
sage: sec2_plot = plot(1/2*x+1/2,xmin=-3,xmax=3,
                    color=pink,linestyle='--')
sage: sec3_plot = plot(x,xmin=-3,xmax=3,
                    color=pink,linestyle='--')
sage: sec4_plot = plot(3/2*x-1/2,xmin=-3,xmax=3,
                    color=pink,linestyle='--')
sage: my_anim = animate((
                    com_plot+sec1_plot, com_plot+sec2_plot,
                    com_plot+sec3_plot, com_plot+sec4_plot,
                    com_plot
                ))
sage: show(my_anim)
```

Notes on `xmin`, `xmax`

- In `plot()`,
`xmin` and `xmax` indicate x values to *compute*.
- In `show()` and `animate()`,
`xmin` and `xmax` indicate x values to *display*.
- `plot(x^2,xmin=-3,xmax=3)` computes points on the interval $[-3,3]$
- `show(my_plot,xmin=-1,xmax=1)` displays $x \in [-1,1]$,
regardless of the x values computed in `my_plot`

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Implicit plots

- plot of an equation in x and y
 - $x^2 + y^2 = 1$
 - might not be a function of x !
- Implicit plots handled by Python package `matplotlib`
- Behave differently than usual plots
 - look different
 - different options

The `implicit_plot()` command

`implicit_plot($f(x,y)$, (x , $xmin$, $xmax$), (y , $ymin$, $ymax$),
options)` where

- $f(x,y)$ a function of x and y
 - graphs $f(x,y) = 0$
 - variable y must be defined!
- *options* include
 - `plot_points`: number of points in each direction
 - `fill` (True or False): fill the region $f(x,y) < 0$
 - `color` (*not* `rgbcolor`)

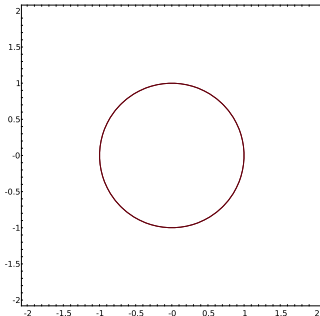
Example

To plot a circle, rewrite the equation as $f(x,y) = 0$:

$$x^2 + y^2 = 1 \implies x^2 + y^2 - 1 = 0 \implies f(x,y) = x^2 + y^2 - 1$$

```
sage: icircle = implicit_plot(x**2 + y**2-1,  
                             (x,-2,2), (y,-2,2), color='red')
```

```
sage: show(icircle, aspect_ratio=1)
```

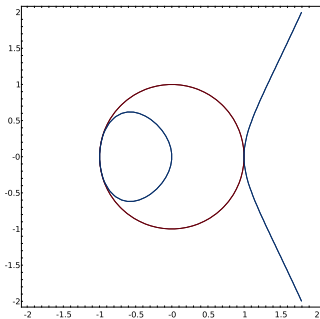


Combining

Can combine implicit plots:

```
sage: ielliptic = implicit_plot(y**2-x**3+x,  
                                (x,-2,2),(y,-2,2),color='blue')
```

```
sage: show(icircle+ielliptic,aspect_ratio=1)
```



Animating

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```
sage: elliptic1 = implicit_plot(y**2-x**3+x,
                                (x,-2,2),(y,-2,2),color='blue',plot_points=100)
sage: elliptic2 = implicit_plot(y**2-x**3+0.75*x,
                                (x,-2,2),(y,-2,2),color='blue',plot_points=100)
sage: elliptic3 = implicit_plot(y**2-x**3+0.5*x,
                                (x,-2,2),(y,-2,2),color='blue',plot_points=100)
sage: elliptic4 = implicit_plot(y**2-x**3+0.25*x,
                                (x,-2,2),(y,-2,2),color='blue',plot_points=100)
sage: elliptic5 = implicit_plot(y**2-x**3+0.1*x,
                                (x,-2,2),(y,-2,2),color='blue',plot_points=100)
sage: elliptic6 = implicit_plot(y**2-x**3,
                                (x,-2,2),(y,-2,2),color='blue',plot_points=100)
sage: my_anim = animate([elliptic1, elliptic2, elliptic3,
                           elliptic4, elliptic5, elliptic6],
                           aspect_ratio=1)
sage: show(my_anim)
```

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Parametric equations

Form:

$$\begin{cases} x(t) = \dots \\ y(t) = \dots \end{cases}, \quad t \in [t_{\min}, t_{\max}]$$

Example

A Bezier curve w/control pts (x_0, y_0) , (x_1, y_1) , (x_2, y_2) , (x_3, y_3)

$$\begin{cases} x(t) = x_0(1-t)^3 + x_1t(1-t)^2 + x_2t^2(1-t) + x_3t^3 \\ y(t) = y_0(1-t)^3 + y_1t(1-t)^2 + y_2t^2(1-t) + y_3t^3 \end{cases}, \quad t \in [0, 1].$$

The `parametric_plot()` command

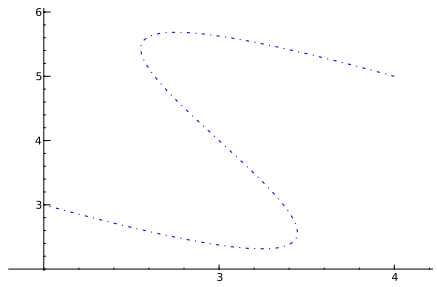
`parametric_plot((x(t),y(t)), (tmin,tmax), plot options)`

where

- $x(t), y(t)$ functions of t
- don't forget to define t as a variable
- usual plot options apply

Example Bezier Curve

```
sage: parametric_plot(  
      (2*t**3 + 6*3*t**2*(1 - t)  
       + 0*3*t*(1 - t)**2 + 4*(1 - t)**3,  
       3*t**3 + 0*3*t**2*(1 - t)  
       + 8*3*t*(1 - t)**2 + 5*(1 - t)**3),  
      (0,1),linestyle='-.')
```



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Polar plots

Form: r (radius) a function of θ (angle)

Example

A limaçon has the form

$$r = 1 + c \sin \theta.$$

It is difficult to describe this implicitly.

The `polar_plot()` command

`polar_plot($r(x)$, options)` where

- $r(x)$ is a *polar* function of x
 - x stands in for θ
 - can define a variable θ if you really want, but...
- usual plot options apply

Example limaçon

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```
sage: lim1 = polar_plot(1+2.5*sin(x),xmin=0,xmax=2*pi)
sage: lim2 = polar_plot(1+1.7*sin(x),xmin=0,xmax=2*pi)
sage: lim3 = polar_plot(1+sin(x),xmin=0,xmax=2*pi)
sage: lim4 = polar_plot(1+0.7*sin(x),xmin=0,xmax=2*pi)
sage: lim5 = polar_plot(1+0.5*sin(x),xmin=0,xmax=2*pi)
sage: lim6 = polar_plot(1+0*sin(x),xmin=0,xmax=2*pi)
sage: my_anim = animate([lim1,lim2,lim3,
                        lim4,lim5,lim6],
                        aspect_ratio=1,xmin=-2,xmax=2,
                        ymin=-1,ymax=4)
sage: show(my_anim)
```

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The `.save()` command

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Summary

Images and animations can be saved and used by other programs:

```
sage: my_plot = plot(x**2, -3, 3)
```

```
sage: my_plot.save('a_parabola.pdf')
```

The `.save()` command

Images and animations can be saved and used by other programs:

```
sage: my_plot = plot(x**2, -3, 3)
```

```
sage: my_plot.save('a_parabola.pdf')
```

Can now

- open `a_parabola.pdf` in PDF reader (Adobe's Reader, Apple's Preview, Gnome's `evince`, KDE's `okular`, etc)
- import `a_parabola.pdf` into word processor (Microsoft Office, OpenOffice, Lyx, etc)
- submit it to your instructor

Image formats

Different formats available

- bitmap
 - colored dots
 - does not scale well
 - gif, jpg, png
- vector
 - mathematical instructions
 - scales to any size
 - eps, pdf, svg

Example

- ① When viewing previous image, zoom in & out
 - quality does not degenerate
- ② Now try this:
`sage: my_plot.save('a_parabola.png')`
 - open in viewer, zoom in
 - notice blockiness

Saving animation

- can save only as gif's
`sage: my_anim.save('limacon.gif')`
- open in Firefox or other web browser

Outline

- 1 Basic 2-D objects
- 2 Plotting functions
- 3 Options for displaying plots
- 4 Implicit plots
- 5 Parametric and polar plots
- 6 Saving images and animations
- 7 Summary

Summary

- Sage offers many commands for plotting 2-D objects
 - points, lines
 - functions
 - equations: implicit, parametric, polar
- Most options work for all objects
- Combine objects by “adding” them together
- Animate using a list of objects
- save images