MAT 305: Review #5

June 12, 2014

Directions: The usual counsels apply.

1. Create a new worksheet. Set the title to, "Review #5". Add other information to identify you, as necessary.

2. Select a problem according to the following schema.

If your ID ends with...	...use this function...	...over this interval...	...with $a =$...
1,2,3,4	$f(x) = e^x$	$[0, 1]$	$1/2$
5,6,7,8	$f(x) = \ln x$	$[1, e]$	2
other	weird: see me		

Part 1: Derivatives

3. Find the equation of the line tangent to f at $x = a$. Any computation that *can* be done with Sage should be evident in your worksheet!

4. Combine the plots of both f and the tangent line over the interval given. The curve for f should be black, and have a width of 2. The line should be blue, and have a width of 2.

5. Create an animation with at least 8 frames that shows the approach of the secant line to the tangent line as $x \to a$ from the left. Reuse the plots of f and the tangent line from above. The secant lines should be red, and have a width of 1. You are free to choose any points you like for the secant, just so long as $x \to a$ from the left. When you are done, your animation should resemble the one on the course syllabus: for instance, the secant line should proceed back and forth, not just in one direction.

Part 2: Exact integrals

6. Compute the *net* area between f and $g(x) = 1 - x^2$ over the interval given. (In case you've forgotten, "net" area means you don't have to worry about intersections.)

7. Combine the plots of both f and g over the interval given. Fill in the area between f and g. The curves for both f and g should be black, with a width of 2. The filling can be any color you like, except black. Also, make the fill half-transparent. Add a text label inside the filling which contains the area.

Part 3: Approximate integrals

8. Go back to your Calculus text and review the calculation of arclength with integrals. Write the formula (use LaTeX!), and in a text cell explain briefly what geometric tool is used to approximate it.

9. Use Sage to approximate the arclength of the ellipse $x^2/4 + y^2/9 = 1$. Limit the approximation to 5 sample points, and round your answer to 5 decimal places.

10. Repeat problem 9, this time limiting the approximation to 10 sample points. What part of the answer indicates that you have a more accurate answer?

Part 4: BONUS! (for those with exceptional time and/or motivation)

Animate approximations of the arclength, where each frame shows

- no axes;
- the ellipse, in black, of width 2;
- 5, 6, 7, . . . , 10 dashed line segments, in red, of width 1;
- a text label with the corresponding approximation to the arclength, at the center of the ellipse, in black.

This bonus is worth as much as *the entire assignment*. If you wish, you may do Part 4 instead of Parts 1–3. Be sure you know what you are doing; this can take a while.