MAT 305: Review #3

June 5, 2014

Remark. In this assignment, we view all solutions as complex numbers $a + bi$, where $i^2 = -1$. When you are asked to "plot $a + bi$ on the complex plane," plot it as the point (a, b).

1. Create a new worksheet. Set the title to, "Review #3". Add other information to identify you, as necessary.
2. Use Sage to solve the equation $x^2 - 1 = 0$. Plot the solutions on the complex plane. It's not very interesting, is it?
3. Use Sage to solve the equation $x^3 - 1 = 0$. Plot the solutions on the complex plane. This is a little more interesting.
4. Use Sage to solve the equation $x^4 - 1 = 0$. Plot the solutions on the complex plane. This might be a little boring, again.
5. Use Sage to solve the equation $x^5 - 1 = 0$. Plot the solutions on the complex plane. This figure should be arresting, especially if you connect the dots, but don't do that; just plot the points, note the result, and move on.

   Extra Credit: Use a `sum()` of points generated using a `for` loop *in* a collection, to the effect of

   ```
   sum([point(...)  for ...])
   ```

6. Can you guess? Yep, use Sage to solve the equation $x^6 - 1 = 0$. Plot the solutions on the complex plane. This is probably getting boring, if only because you can probably guess the result not only of this one, but also of...
7. Use Sage to solve the equation $x^7 - 1 = 0$. Plot the solutions on the complex plane. If you understand the geometric pattern, move on; if not, complain. Or, if you want to save time, you can probably guess that I'll tell you to plot the solutions for a few more examples of $x^n - 1 = 0$, only with larger values of n. Sooner or later, you'll perceive a geometric pattern.
8. To each of the previous plots, add the parametric plot of $\cos(2\pi t) + i\sin(2\pi t)$ for $t \in [0, 1]$, remembering as before that you plot $a + bi$ as (a, b). In a text cell, explain why the geometric pattern of the images justify the assertion that the solutions to $x^n - 1$ all have the form

   $$\cos\frac{2\pi k}{n} + i\sin\frac{2\pi k}{n}.$$

   For full credit, explain how each value of k and n relates to each solution.

9. Time to up our game.[1] Experiment with $x^n - a$ for some nice values of a and a fixed, but sufficiently large, value of n, such as $n = 5$. What do you see? Formulate a conjecture as to the form of these solutions.

10. As before, make it all look nice, with sectioning, commentary in text boxes, at least a *little* LaTeX, etc. I'm happy to help with LaTeX if you need it, as with anything else, though if it's something you can look up without too much trouble, I'll tell you to do that.

[1] I'd rather say, "Let's generalize," as such cliches don't fit my personality one whit, but They tell me it reaches the students better. (Don't ask who They are.)