

INDIVIDUAL ASSIGNMENT 1

MAT 305 FALL 2011

If you view the class syllabus online, you notice an animation at the top of the page. This animation shows a relationship that you learned in Calculus:

$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \left(\frac{\Delta y}{\Delta x} \right);$$

that is, the instantaneous rate of change of y , also called its **derivative**, is the limit of the average rates of change of y between x_1 and x_2 as $\Delta x = x_2 - x_1$ approaches zero.

Rely on what you know from Calculus to answer 1–3; answer the rest and submit *all* work as a **Sage worksheet** on <https://pax.st.usm.edu:8004/>.

- (1) Describe how the animation represents the instantaneous rate of change of y .
- (2) Describe how the animation represents the average rates of change of y .
- (3) Describe how the animation represents the relationship between the average rates of change and the instantaneous rate of change.
- (4) Use the last digit of your student number to select the following function and point.

if your student number ends with... let $f(x) = \dots$ and $a = \dots$

0,1	$e^{-x} \cos x$	$-\frac{\pi}{4}$
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2,3	$e^{-x} \sin x$	$\frac{\pi}{4}$
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4,5	$\ln(1 + x^2)$	0
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6,7	$\frac{1}{1 + x^2}$	0
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8,9	$\frac{2x}{1 + x^2}$	1
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- (5) Plot f in black over a small neighborhood of $x = a$.
- (6) Show that the derivative of f at $x = a$ is 0.
Note: Use Sage to do this; don't do it by hand.
- (7) Plot both f and the line tangent to f at $x = a$. Make the tangent line blue.
- (8) Choose four x values $b_1, b_2, b_3,$ and b_4 close to $x = a$. Compute the slopes of the secant lines between a and b_i for $i = 1, 2, 3, 4$.
- (9) Create four plots, each of which combines f with a blue secant line.
- (10) Combine all the plots to obtain an animation similar to the one on my webpage.
Hint: Notice that on the webpage the secant lines move forwards and backwards; your animation should replicate this behavior.