MAT 305: Mathematical Computing

Functions in computer programming

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Fall 2011

Outline

Outline

Functions?

- **function:** a sequence of statements organized as one command
  - may return one or more values
- names in other languages
  - C family: “functions”
  - Pascal family: “procedures” (no result) or “functions” (result)
  - object-oriented languages: “methods” or “features”

Why functions?

- avoid retyping code
  - many patterns repeated
  - same behavior, different data
- organization, abstraction
- easier to read, maintain

Defining a function

```
def name( argument1=default1 , argument2=default2 , ... ):
    statement1
    statement2
    ...
```

where

- *name* is an identifier
- *arguments* (optional) are identifiers
- *defaults* (optional) are default values for the corresponding arguments

Defining a function

`def` *name*`(` *argument1=default1* `,` *argument2=default2* `, ...)`:
  statement1
  statement2
  ...

where

- *name* is an identifier
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***not** optional:*

- `:`, `()`, `def`
- *at least one statement*
- *indent all statements* in function

Calling a function

- once f is defined, call using `f()`
- supply data for arguments without default values

Example

```
def main():
  print 'Hello, world'
```

- name of function is `main`

  *terrible choice; **do not use***

- no arguments
- one statement

Example

Try it!

```
sage:  def main():
         print 'Hello, world'
sage:  main()
Hello, world
```

Outline

Arguments?

argument: a placeholder for data

scope: name visible only inside function where it is defined

- data still exists outside function
- modifying argument does not modify original data, but creates new data
  - *caveat:* contents of lists and sets can be modified
- value of data forgotten immediately after function concludes

Example

```
def hello(name='world'):
  print 'Hello,', name
```

- name of function is `hello`
- one argument, `name`
  - default value: `'world'`

Example

Try it!

```
sage:  def hello(name='world'):
         print 'Hello,', name

sage:  hello()
Hello, world
```

Example

Try it!

```
sage:  def hello(name='world'):
         print 'Hello,', name

sage:  hello()
Hello, world

sage:  hello('Pythagoras')
Hello, Pythagoras
```

Example

Try it!

```
sage:  def hello(name='world'):
         print 'Hello,', name

sage:  hello()
Hello, world

sage:  hello('Pythagoras')
Hello, Pythagoras

sage:  hello(pi)
Hello, pi
```

Warning 1

Don't use uninitialized identifiers

```
sage:  def hello(name='world'):
         print 'Hello,', name
```

```
sage:  hello(Pythagoras)
```

oops: no quotes!

...*Output deleted*...

```
NameError:  name 'Pythagoras' is not defined
```

Warning 2

Scope implies `name` does not exist outside `hello`

```
sage:  def hello(name='world'):
         print 'Hello,', name

sage:  hello('Pythagoras')
Hello, Pythagoras

sage:  name
'KodairaSymbol'
```

???

Warning 3

Scope implies `name` forgotten once `hello` concludes

```
sage:  def hello(name='world'):
         print 'Hello,', name

sage:  hello('Pythagoras')
Hello, Pythagoras

sage:  hello()
Hello, world
```

name has value `'world'` *again*

Warning 4

Can change value inside function, but value outside function remains the same

```
sage:  def mischievous_hello(name='world'):
           name = 'loser!'
           print 'Hello,', name
```

Warning 4

Can change value inside function, but value outside function remains the same

```
sage:  def mischievous_hello(name='world'):
         name = 'loser!'
         print 'Hello,', name

sage:  print_name = 'Dr.  Perry'
sage:  mischievous_hello(print_name)
Hello, loser!
```

value of `name` *changed in function*

Warning 4

Can change value inside function, but value outside function remains the same

```
sage:  def mischievous_hello(name='world'):
         name = 'loser!'
         print 'Hello,', name

sage:  print_name = 'Dr.  Perry'
sage:  mischievous_hello(print_name)
Hello, loser!                    value of name changed in function

sage:  print_name
'Dr.  Perry'                     value of print_name unchanged
```

Warning 5

If defaults are not given to arguments, you must supply something

```
sage:  def hello(name='world'):
         print 'Hello,', name
sage:  def goodbye(name):
         print 'Goodbye,', name
```

no default for `name`

Warning 5

If defaults are not given to arguments, you must supply something

```
sage:  def hello(name='world'):
         print 'Hello,', name
sage:  def goodbye(name):
         print 'Goodbye,', name
sage:  hello()
Hello, world
```

no default for `name`

Warning 5

If defaults are not given to arguments, you must supply something

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         print 'Hello,', name
sage:  def goodbye(name):
         print 'Goodbye,', name
```

no default for `name`

```
sage:  hello()
Hello, world
sage:  goodbye()
```

...*Output deleted*...

```
TypeError:  goodbye() takes exactly 1 argument (0
given)
```

Warning 5

If defaults are not given to arguments, you must supply something

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sage:  def hello(name='world'):
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sage:  def goodbye(name):
         print 'Goodbye,', name
```

no default for `name`

```
sage:  hello()
Hello, world
sage:  goodbye()
```

...*Output deleted*...

```
TypeError:  goodbye() takes exactly 1 argument (0
given)
sage:  goodbye('cruel world')
Goodbye, cruel world
```

Arguments, lists and sets

- Function does not change the value of an argument outside function
- However, if argument is a mutable collection C:
  - C cannot be changed, but
  - *elements* of C can be changed

Example: C does not change

```
sage:  def modify_C(C):
           C = [0,1,2,3]

sage:  L = [-1,0,1]

sage:  modify_C(L)

sage:  L
[-1, 0, 1]
```

Example: elements of C change

```
sage:  def modify_els_of_C(C):
           C[0] = 0

sage:  L = [-1,0,1]

sage:  modify_els_of_C(L)

sage:  L
[0, 0, 1]
```

Why does this happen?

Hand-waving / Lawyer's argument

- `L` is a list of 3 elements
  - data does not change
  - function concludes: `L` is still a list of 3 elements
- `L[0]`, `L[1]`, `L[2]` are *elements* of `L`
  - these data **are not** "arguments" to function
  - $\therefore$ can be changed

Why does this happen?

Analogy: defacing library books doesn't change catalog

- `L` is address of a location in memory
  - similar to library's reference number for book

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- `L` is address of a location in memory
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- Python *copies* `L`'s value
  - write reference number on a scrap sheet of paper
  - original reference still in catalog

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Analogy: defacing library books doesn't change catalog

- `L` is address of a location in memory
  - similar to library's reference number for book
- Python *copies* `L`'s value
  - write reference number on a scrap sheet of paper
  - original reference still in catalog
- Function learns (and cannot change) `L`'s value, *but*...
  - can deface book at that location, *even though*
  - changing number on scrap sheet of paper (`C`) doesn't change catalog entry (`L`)
  - $\therefore$ function can change information at location

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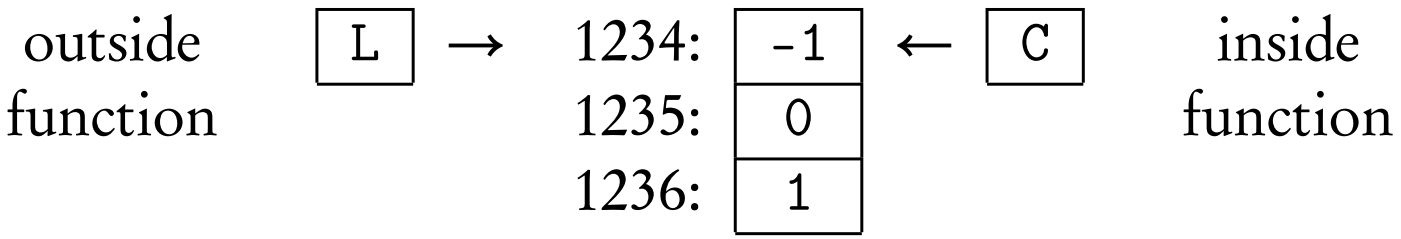
- Function learns (and cannot change) `L`'s value, *but*...
  - can deface book at that location, *even though*
  - changing number on scrap sheet of paper (`C`) doesn't change catalog entry (`L`)
  - $\therefore$ function can change information at location
- Function concludes: data changed but `L` unchanged
  - books defaced, but catalog still references them

Why does this happen?

Precise answer: lists are pointers



- List @ location 1234 $\Longrightarrow$ `L` $\longrightarrow$ `1234`

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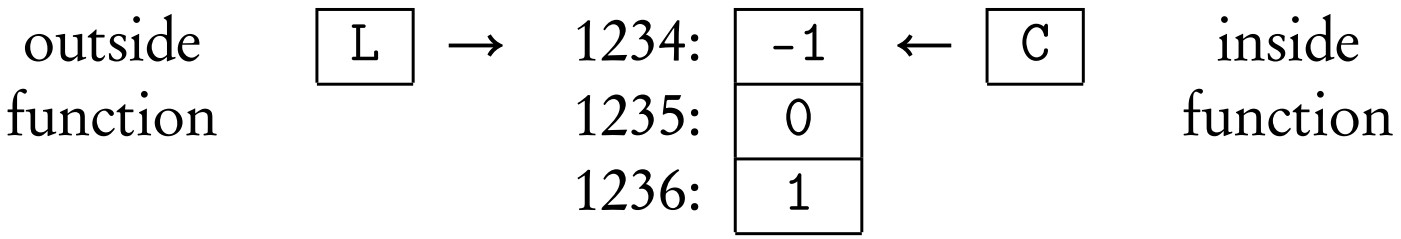
Precise answer: lists are pointers



- List @ location 1234 $\Longrightarrow$ `L` $\longrightarrow$ `1234`
- $\therefore$ `C` $\longrightarrow$ `1234`
- Function now has access to memory *at* `L`
  - changing `C` won't change `L`
  - changing `C[0]` changes `L[0]`

Outline

Returning values?

- Functions compute values
- Often want to work with what we've computed
  - `print` command not very helpful

Example

Compute derivative of special function, want to use it to:

- graph tangent line
- analyze concavity
- identify optimum values
- ...

The `return` command

`return` *value1*, *value2*, ...

- reports the data values *value1*, *value2*, etc.
- only works inside functions
- only reports to caller of current function

Example problem

Write a program to compute the line tangent to $f(x)$ at $x = x_0$.

- Write pseudocode answering:
  1. What inputs will we need?
     - domain of each input (what set/type of object)
  2. What outputs do we expect?
     - inputs' purpose & relationship to output
  3. How do we use the inputs to generate the output?
     - think step-by-step
     - do a sample problem: $f(x) = x^2$, $x_0 = 3$
     - think about possible errors errors
- Implement pseudocode

Pseudocode?

description of algorithm

- many formats
- format independent of computer language
- prefer mathematics to programming
  - "ith element of L" or "L_i", not `L[i-1]`

Our pseudocode format

algorithm *name*

inputs

input1 $\in$ *domain1*
input2, *description of type*
...

outputs

output1, *relationship to inputs*
output2, *relationship to inputs*
...

do

English or mathematical statement 1
English or mathematical statement 2
...

Our pseudocode format

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do

English or mathematical statement 1
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Try it now on the given problem

Example pseudocode

algorithm *tangent_line*

inputs

f, a function of a variable x

$x_0 \in \mathbb{R}$

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— *We need two things for a line: a point* (x_0, y_0) *and the slope* m

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Let $y_0 = f(x_0)$

— *Use Calculus to find* m

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Let $y_0 = f(x_0)$

— *Use Calculus to find* m

Let $fderiv = f'(x)$

Let $m = fderiv(x_0)$

Example pseudocode

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Let $y_0 = f(x_0)$

— *Use Calculus to find* m

Let $fderiv = f'(x)$

Let $m = fderiv(x_0)$

— *Point-slope form:* $y - y_0 = m(x - x_0)$

Let $line = m(x - x_0) + y_0$

Example pseudocode

algorithm *tangent_line*

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Let $y_0 = f(x_0)$

— *Use Calculus to find* m

Let $fderiv = f'(x)$

Let $m = fderiv(x_0)$

— *Point-slope form:* $y - y_0 = m(x - x_0)$

Let $line = m(x - x_0) + y_0$

return *line*

Example implementation

```
def tangent_line(f, x, x0):
  # returns the line tangent to f at x=x0
  y0 = f(x=x0)
  df = diff(f,x)
  m = df(x=x0)
  tanline = m*(x - x0) + y0
  return tanline
```

Example implementation

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  # returns the line tangent to f at x=x0
  y0 = f(x=x0)
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  return tanline
```

Why have x as an input? Many reasons:

- $f(t)$
- other variables in function

so let's specify variable as well

Comments

Notice line that begins with # (--- in psuedocode)

- Sage *ignores* anything after this symbol
- Use to explain intent to reader
    - you, too, are reader!

Example run

```
sage:  def tangent_line(f, x, x0):
         # returns the line tangent to f at x=x0
         y0 = f(x=x0)
         df = diff(f,x)
         m = df(x=x0)
         tanline = m*(x - x0) + y0
         return tanline
sage:  tangent_line(x**2, x, 1)
2*x - 1
```

Example run

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         # returns the line tangent to f at x=x0
         y0 = f(x=x0)
         df = diff(f,x)
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         tanline = m*(x - x0) + y0
         return tanline
sage:  tangent_line(x**2, x, 1)
2*x - 1
```

Behold: *the power of symbolic computation!*

```
sage:  var('a b c')
sage:  tangent_line(a*x**2 + b*x + c,x,1)
(x - 1)*(2*a + b) + a + b + c
```

Combine with plots

We show the plots of e^x and its tangent line at $x = 0$

```
sage:  f = e**x
sage:  tanline = tangent_line(f, x, 0)
sage:  fplot = plot(f,-2,2,rgbcolor='black',
                    thickness=2)
sage:  lineplot = plot(tanline,-2,2,linestyle='--')
sage:  fplot + lineplot
```

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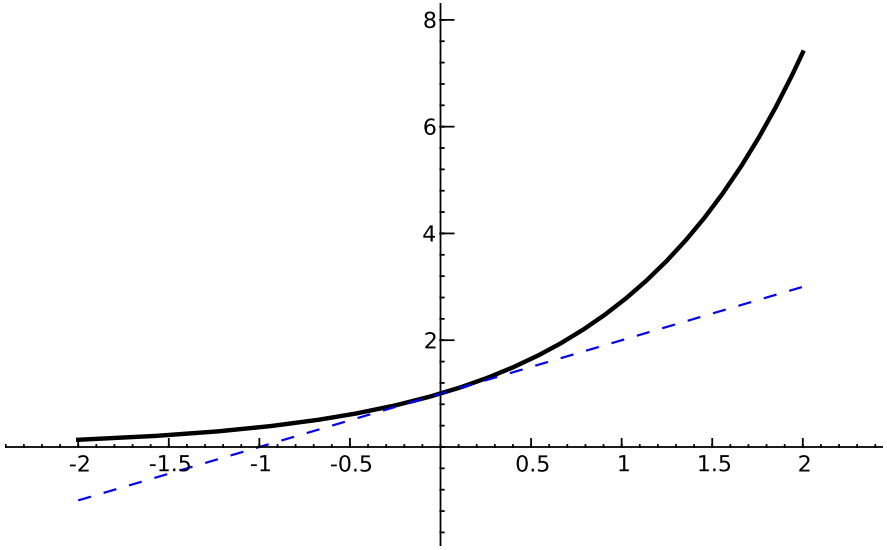

Combining functions

It would be nice to have a function that graphs an arbitrary $f(x)$ and its tangent line at $x = x_0$. Options include:

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- Repeat previous commands for each f and each x_0

a lot of work!

Combining functions

It would be nice to have a function that graphs an arbitrary $f(x)$ and its tangent line at $x = x_0$. Options include:

- Repeat previous commands for each f and each x_0

  a lot of work!

- Encapsulate commands in another function

Pseudocode

algorithm *plot_function_and_tangent*

inputs

f, a function of a variable x

$x_0 \in \mathbb{R}$

outputs

the plot of $f(x)$ and the line tangent to f at $x = x_0$

do

Let P_1 be the plot of $f(x)$ in a neighborhood of x_0

Let $g(x)$ be the line tangent to f at x_0 *Already solved!*

Let P_2 be the plot of $g(x)$ in the same neighborhood of x_0

return P_1 and P_2 combined

Implementation

```
def plot_function_and_tangent(f, x, x0=0,
      xmin=-2, xmax=2):
```

Whitespace

```
  # plots f(x) and line tangent to f at x0
  # over [ xmin, xmax ];
  # returns combination of these plots
```

distinguishes

```
  P1 = plot(f, xmin, xmax, rgbcolor='black',
        thickness=2)
```

different

```
  # next line reuses previous code
  g = tangent_line(f, x, x0)
```

tasks

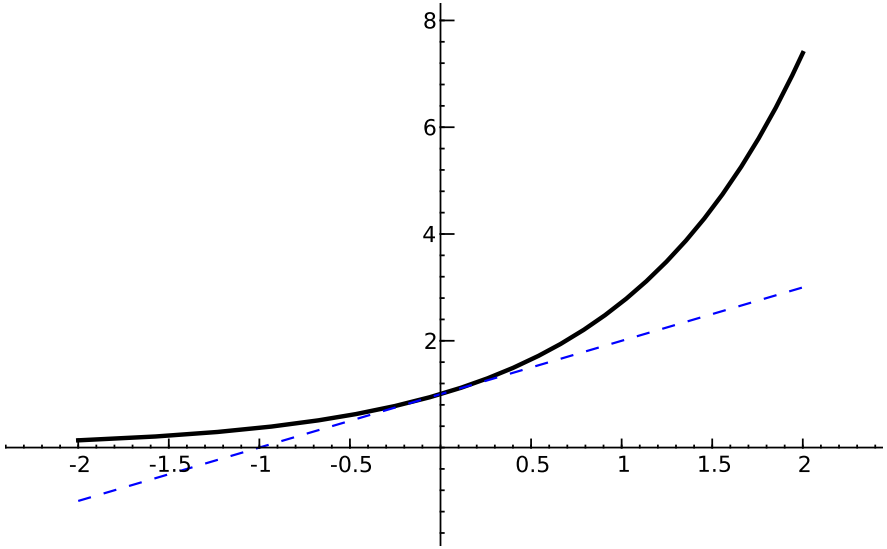
```
  P2 = plot(g, xmin, xmax, linestyle='--')
  return P1 + P2
```

Examples

```
sage:  def plot_function_and_tangent...
```

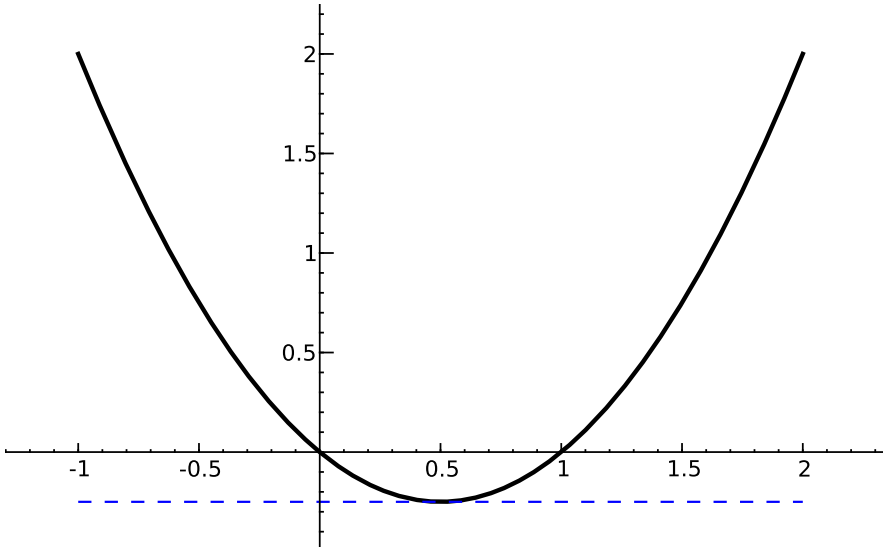
Examples

```
sage:  def plot_function_and_tangent...
sage:  plot_function_and_tangent(e**x,x)
```



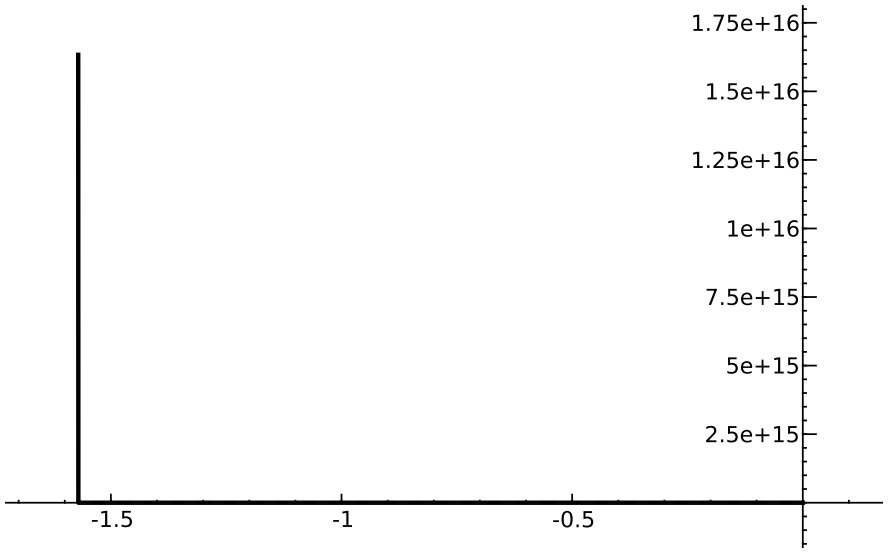
Examples

```
sage:  def plot_function_and_tangent...
sage:  plot_function_and_tangent(x**2-x,x,xmin=-1,
         x0=0.5,xmax=2)
```



Examples

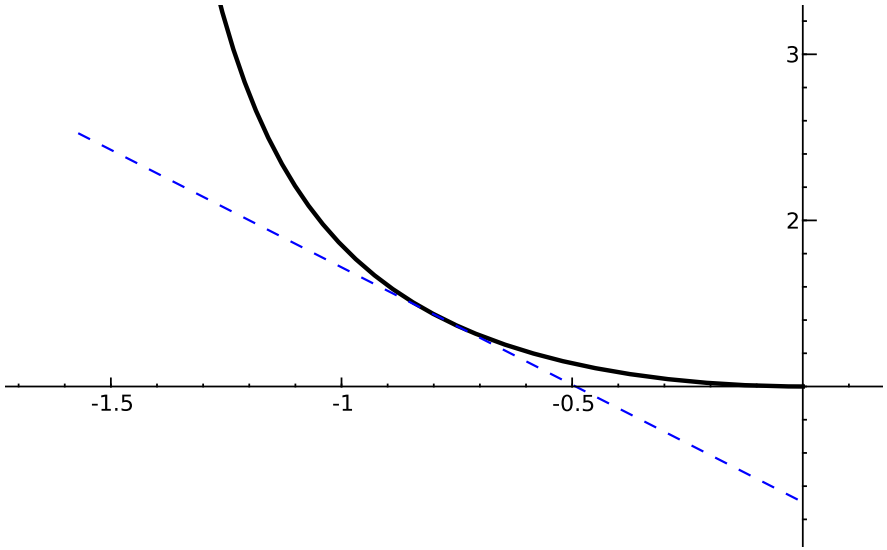
```
sage:  def plot_function_and_tangent...
sage:  plot_function_and_tangent(sec(x),x,
         xmin=-pi/2,x0=-pi/4,xmax=0)
```



ouch. need to adjust `ymax`

Examples

```
sage:  def plot_function_and_tangent...
sage:  good_sec = plot_function_and_tangent(
         sec(x),x,
         xmin=-pi/2,x0=-pi/4,xmax=0)
sage:  show(good_sec,ymax=3)
```



Note: `sec(x)` *does not work in older versions, apparently because its derivative is not computed*

Outline

Summary

- Functions collect several commands into one
  - organizes solutions to problems
  - abstraction makes problem-solving easier
- define using `def(...):`
- Functions receive *arguments* as data
  - can specify default values
  - function does not change arguments, *but*...
  - elements of collections can be changed
- Return value(s) using `return`