## TEAM ASSIGNMENT 3

MAT 305 FALL 2009

## 1. Directions

The groups for this assignment are

| Group 1 | Group 2 | Group 3 |
| :---: | :---: | :---: |
| Names deleted |  |  |
|  |  |  |
| Group 4 |  | Group 5 |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

Solve the following problems in a Sage worksheet. Each group should submit one worksheet. Submit the worksheet by sharing the worksheet with my account (mat305_fa2009) at
https://sage.st.usm.edu:8000/

If you wish, you may share with all the members of the group who are registered at that website, but this is not necessary. Remember to list all team names on the worksheet, or I will deduct 10\% for inconveniencing me to look up your names.

Submitted worksheets should include comments at the beginning of each function indicating the function's purpose, the meaning of the inputs, the intended output, and an example invocation.

The due date for this assignment is

## 2. The Assignment

1. Write a Sage interact that demonstrates:

The Mean Value Theorem of Derivatives. For any function $f$ that is differentiable over the closed interval $[a, b]$, there exists some $c \in(a, b)$ such that the slope of the tangent line at $x=c$ is precisely the slope of the secant line connecting $(a, f(a))$ and $(b, f(b))$. That is,

$$
f^{\prime}(c)=\frac{f(b)-f(a)}{b-a}
$$

The Sage interact should:

- Offer the user input boxes for a function $f$ and endpoints $a$ and $b$ of the interval.
- Compute $f^{\prime}(x)$.
- Compute $m=\frac{f(b)-f(a)}{b-a}$.
- Use the code one of your teams developed to implement Newton's Method to approximate a root of the equation $f^{\prime}(x)=m$ for $x$, thus computing $c$.
- Combine the following plots that represent the Theorem:
- a plot of $f(x)$ over the interval $[a, b]$;
- points at $(a, f(a)),(b, f(b))$, and $(c, f(c))$;
- the secant line joining $(a, f(a))$ and $(b, f(b))$; and
- the tangent line at $(c, f(c))$.

If all goes well, the two lines displayed should be parallel. You do not need to supply pseudocode for your function.
2. [from the old textbook, p. 138 Ex. 7] Suppose $f$ is a function of two variables defined on a rectangle $R=[a, b] \times[c, d]$ in the plane. Write a function to approximate the minimum and maximum values of $f$ on $R$ and return the corresponding points. (Pseudocode appears on the following page.) Write another function that produces a 3d plot of $f$ on $R$, and vertical line segments connecting the points $\left(x_{\min / \max }, y_{\min / \max }, 0\right)$ and $\left(x_{\min / \max }, y_{\min / \max }, z_{\min / \max }\right)$. Test your functions using

- $f(x, y)=x e^{-x^{2}-y^{2}}$ over $[-2,2] \times[-2,2]$, and
- $g(x, y)=5(x+y) /\left(1+x^{2}+y^{2}\right)$ over $[-5,5] \times[-5,5]$.

3. (Optional Bonus: counts as a free assignment in any category you choose!) In my office, I have a paper published in a recent issue of College Math Journal (or maybe Math Magazine, I forget) on "Sledge-Hammer Integration". Come to my office, read the paper, and implement SledgeHammer Integration in Sage.
```
algorithm optimize_2d
inputs
    \(f\), a function in \(x\) and \(y\)
    - You have a number of choices on how to implement the input \(R\)
    - There is no one right way as long as you document it well
    \(R=[a, b] \times[c, d] \subset \mathbb{R}^{2}\)
    \(M \in \mathbb{N}\), the number of subintervals into which we divide the \(x\)-axis
    \(N \in \mathbb{N}\), the number of subintervals into which we divide the \(y\)-axis
outputs
    \((a, b, c)\), a point that is a maximum or minimum value of \(f\) over \(R\)
do
    - Setup a mesh consisting of the corners of the subregions of \(R\)
    - You probably want to use lists, or something of the sort, for \(x_{i}\) and \(y_{i}\)
    Let \(\Delta x=\frac{b-a}{M}\)
    Let \(\Delta y=\frac{d-c}{N}\)
    for \(i \in\{0, \ldots, M\}\) do
        Let \(x_{i}=a+i \Delta x\)
    for \(i \in\{0, \ldots, N\}\) do
        Let \(y_{i}=c+i \Delta y\)
    - Setup for finding max \& min: use Sage’s keywords -infinity, +infinity
    Let \(z_{\text {max }}=-\infty\)
    Let \(z_{\text {min }}=\infty\)
    - Approximate the min and max by evaluating \(y\) values along the mesh
    for \(i \in\{0, \ldots, M\}\) do
        for \(j \in\{0, \ldots, N\}\) do
            \(z=f\left(x_{i}, y_{j}\right)\)
            if \(z<z_{\text {min }}\) then
                Let \(z_{\text {min }}=z\)
            Let \(\min =\left(x_{i}, y_{j}, z\right)\)
            else if \(z>z_{\max }\) then
                    Let \(z_{\text {max }}=z\)
                    Let \(\max =\left(x_{i}, y_{j}, z\right)\)
    return min, max
```

