INDIVIDUAL ASSIGNMENT 3

MAT 305 FALL 2009

Due date: 1 Dec 2009

For each problem, devise properly-formatted pseducode as well as Python code. Submit your solutions as Sage worksheets, sharing with `mat305_fa2009`. Be sure that your name appears somewhere on the worksheet. Where appropriate, split the task into subtasks and write separate psuedocode and Python code for each subtasks. Each Python-coded function must include at least one comment indicating the purpose of the function.

1. Write a non-recursive function to calculate $n!$, where n is a positive integer. The function must accept n as an input.
2. Write a function to compute the Riemann sum approximation of $\int_a^b f(x)\ dx$ using the Midpoint Rule: For the i-th rectangle on the interval $\left[x_{i-1}, x_i\right]$, use the value $f\left(m_i\right)$ at the midpoint $m_i = \left(x_{i-1} + x_i\right)/2$, for the height of the rectangle. The function must accept as input a function f, the number of partitions N, and the endpoints of the interval a and b.
3. Write an interactive function to compute the Riemann sum approximation of $\int_a^b f(x)\ dx$ using the Midpoint Rule. This interactive function should accept values of f, a, and b in input boxes, and N on a slider from 1 to 10. It should plot f and the rectangles used to approximate the integral, then print the approximate area. The rectangles should appear in a different color with some translucence so as to see the plot of f, but you need not include a color selector. You can either print the approximate area below the graph, or (bonus!) display it as a text object inside the plot.
4. For a point $P = (a, f(a))$ on the graph of f, the *normal line* at P is the line through P which is perpendicular to the tangent line at P. Recall that two lines are perpendicular if and only if their slopes are negative reciprocals of each other. Since the slope of the tangent line is $m = f'(a)$, the slope of the normal line is $-1/m = -1/f'(a)$. Thus the equation for the normal line at $(a, f(a))$ is
$$y = f(a) - \frac{1}{f'(a)}(x - a).$$
In the exceptional case when $f'(a) = 0$ (i.e., the tangent line is horizontal), the normal line is vertical and has equation $x = a$.

   In analogy with the sequence of secant lines, write a function to plot a sequence of five normal lines N_i to the graph of f at points $\left(a + h_i, f\left(a + h_i\right)\right)$, $i = 1, \ldots, 5$, with $h_1 > \cdots > h_5 > 0$ and h_5 small. Also plot the normal line N at $(a, f(a))$. By limiting the domains/ranges in the `plot` and `show` commands, choose suitable lengths for the normal lines N_i so that (i) they all intersect the normal line N, (ii) they start at the point $\left(a + h_i, f\left(a + h_i\right)\right)$ on the curve, and (iii) they are not so long as to make the graph of f too small. Plot the graph of f (on the given interval $[c, b]$) and all six normal lines in the same picture and clearly label which normal line corresponds to which value of h_i.

(a) $f(x) = \frac{1}{4}\left(x^5 - 2x^3 - 4x^2 + x + 4\right)$, with $a = 0.3$, $c = -1$, $b = 2$.
(b) $f(x) = x^2 e^{-x}$, with $a = -0.3$, $c = -1$, $b = 2$.
(c) $f(x) = \sin(x) - \frac{1}{2}\cos 3x$, with $a = 1$, $c = 0.2$, $b = 2$.