DEFINITIONS/BIG-TIME FACTS TO KNOW FOR TEST 1

CALCULUS II

DEFINITIONS

line tangent to $f(x)$ at $x=a$

(geometric) a line that intersects the curve defined by f at $x=a$ *and* goes in the same direction as the curve defined by f

(algebraic) the line $y=m(x-a)+f(a)$ where $m=f'(a)$

derivative of $f(x)$ at $x=a$

(geometric) the slope of the line tangent to f at a

(algebraic) $f'(x)=\dfrac{dy}{dx}=\lim\limits_{\Delta x\to 0}\dfrac{\Delta x}{\Delta y}=\lim\limits_{x\to a}\dfrac{f(x)-f(a)}{x-a}=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

f is continuous on $[a,b]$

(geometric) f is *unbroken:* no gaps, jumps, or asymptotes

(precise) for any $c\in[a,b]$, we can find the limit of f at c by substitution; that is, $\lim\limits_{x\to c}f(x)=f(c)$

f is differentiable on (a,b)

(geometric) f is *smooth:* no corners, cusps, or breaks

(precise) for any $c\in(a,b)$, we can find the derivative of f at c

(definite) integral of $f(x)$ over $[a,b]$

(geometric) the *net* area between the curve of $f(x)$ and the x-axis, starting at $x=a$ and ending at $x=b$ ("net" means it can be negative or zero)

(algebraic) $\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}f(x_i^*)\Delta x$, where $\Delta x = {}^{b-a}/_{n}$ and x_i^* is *any* point in the ith subinterval of width Δx of $[a,b]$, *as long as the limit exists*

Rieman Sum

(geometric) an approximation of the area under a curve using a finite number of rectangles

(algebraic) $\Sigma_{i=1}^{n}f\left(x_i^*\right)\Delta x$, where $\Delta x={}^{(b-a)}/_{n}$ and x_i^* is *any* point in the ith subinterval of width Δx of $[a,b]$

choices of sample point for Riemann sums

right endpoint $x_i^*=a+i\Delta x$

left endpoint $x_i^*=a+(i-1)\Delta x$

midpoint $x_i^*=a+(i-{}^1/_2)\Delta x$

f is **integrable over** $[a,b]$:

(geometric) we can find the area between the curve defined by f over $[a,b]$ and the x-axis

(algebraic) the limit $\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}f(x_i^*)\Delta x$ converges, where x_i^* and Δx are defined as above

indeterminate form: any limit approaching the form ${}^{\pm\infty}/_{\pm\infty}$, ${}^0/_0$, $\infty-\infty$, $0\times\infty$, 0^0, 1^∞, ∞^0

BIG-TIME RESULTS OR TOOLS

Theorem (L'Hôpital's Rule)**.** *If*

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{\pm\infty}{\pm\infty} \qquad \text{or} \qquad \lim_{x\to a}\frac{f(x)}{g(x)}=\frac{0}{0} \quad ,$$

then

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)} \quad .$$

Algorithm (Newton's Method)**.** *Suppose f is differentiable and has a root on (a,b), which we want to approximate to d decimal places.*

(1) *Let $i=0$ and choose a reasonable initial approximation x_0 ;*
(2) *let $x_{i+1}=x_i-\frac{f(x_i)}{f'(x_i)}$;*
(3) *if the first d decimal places of x_i and x_{i+1} agree, then the root is approximately x_i ;*
(4) *otherwise, let $x_i=x_{i+1}$, add 1 to i, and return to step 2.*

EXAMPLE PROBLEMS

This list is by no means exhaustive.

1. Compute the following limits. Use L'Hôpital's Rule *only* if necessary.

$$\lim_{x\to 0}\frac{x+\sin 8x}{x} \qquad \lim_{x\to 0}\frac{x+\sin 8x}{x^2} \qquad \lim_{x\to 0^+}\frac{\sqrt{x}}{\ln x}$$

$$\lim_{x\to\infty}\frac{\sqrt{x}}{\ln x} \qquad \lim_{x\to\infty}\left(x-\sqrt{x^2-9}\right) \qquad \lim_{x\to\infty}\left(x-\sqrt{x^2-9x}\right)$$

$$\lim_{x\to 0^+} x^{1/x} \qquad \lim_{x\to\infty}\left(1+\frac{2}{x}\right)^x$$

2. Find an approximation to the thousandths place for each of the following functions, with the given initial approximation.
   (a) x^2-2, $x=2$
   (b) $\cos x - x$, $x=0$
3. Name two reasons Newton's Method can fail, *even when the function has a root.*
4. Find the indicated area using only geometric methods. *Explain* your computation. Drawing a picture would go a long way towards an explanation, but some words are probably necessary. Merely writing a formula is insufficient!

$$\int_a^b c\,dx \qquad \int_{-3}^{3} x\,dx \qquad \int_2^5 x\,dx \qquad \int_0^6 (3-x)-\sqrt{36-x^2}\,dx$$

5. Approximate the indicated area using three rectangles. Evaluate once using left endpoints, once using right endpoints, and once using midpoints. Compare your answers to the corresponding result in #4, and comment on the results (overestimate, underestimate, etc.).

$$\int_2^5 x\,dx$$

6. Evaluate the indicated area using the definition of the integral and right endpoints as the sample points. Is this an overestimate, an underestimate, or other? Why?

$$\int_0^3 x^2\,dx$$

7. A particle travels at $v(t) = (x-2)^3$ meters/second from $t = 0$ to $t = 4$ seconds.
   (a) Use geometry to explain why the particle's displacement after four seconds is 0.
   (b) Divide the interval $[0,4]$ into four subintervals and use a Riemann sum with midpoints to approximate the particle's displacement. Explain why your approximation is so good in this case.
   (c) Would you have such a good approximation if you used right endpoints instead? Why or why not?
   (d) Use the definition of the integral and right endpoints to evaluate the displacement of the particle after 4 seconds. Notice that your answer should agree with part (a)!