HOMEWORK QUIZ 4 SOLUTIONS

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9. We can use the fact that $3x^8 - 2$ is even (symmetric to the y-axis) to simplify

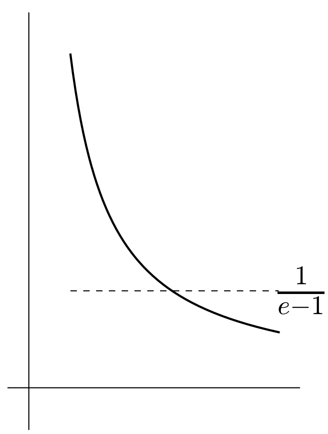
$$\int_{-2}^{2} 3x^8 - 2\,dx = 2\int_{0}^{2} 3x^8 - 2\,dx = 2\left(\frac{x^9}{3} - 2x\right)\Bigg|_0^2 = 2\left(\frac{512}{3} - 4\right) = \frac{1024}{3} - 8 = \frac{1000}{3}.$$

17. We can use use the fact that $\sin x$ is odd (symmetric to the origin) to simplify $\int_{-\pi}^{\pi} \sin x\,dx = 0$.

25. By the Mean Value Theorem, the average value on the interval is

$$\frac{1}{e-1}\int_1^e \frac{1}{x}\,dx = \frac{1}{e-1}\ln|x|\Big|_1^e = \frac{1}{e-1}(\ln e - \ln 1) = \frac{1}{e-1}(1-0) = \frac{1}{e-1}.$$

The problem also asks us to "[d]raw a graph of the function and indicate the average value." (Some of you did not do this. I was lenient on the quiz, but will deduct on the test!) A good graph might look like this:

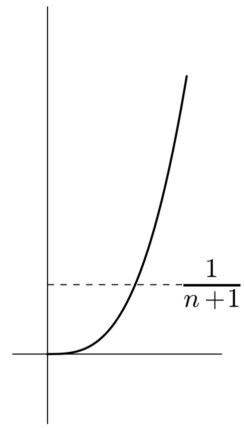


It would not be wrong to draw something like Figures 5.51 and 5.53 on p. 372 either, but they're really asking for something like Figures 5.52 and 5.54.

29. By the Mean Value Theorem, the average value on the interval is

$$\frac{1}{1-0}\int_0^1 x^n\,dx = \frac{x^{n+1}}{n+1}\Bigg|_0^1 = \frac{1}{n+1} - 0 = \frac{1}{n+1}.$$

As in #25, the problem asks for a graph, so I would suggest something like so:



39. The problem asks us to find $c \in (-1,1)$ such that $f(c) = \frac{1}{1-(-1)} \int_{-1}^{1} 1 - |x| \, dx$. I advise dealing with absolute values by splitting them over their domain. The value of the integral is

$$\begin{aligned}
\frac{1}{2}\int_{-1}^{1} 1 - |x| \, dx &= \frac{1}{2}\left[\int_{-1}^{0} 1 - (-x) \, dx + \int_{0}^{1} 1 - x \, dx\right] \\
&= \frac{1}{2}\left[\left(x + \frac{x^2}{2}\right)\Big|_{-1}^{0} + \left(x - \frac{x^2}{2}\right)\Big|_{0}^{1}\right] \\
&= \frac{1}{2}\left[\left(0 - \left(-1 + \frac{1}{2}\right)\right) + \left(\left(1 - \frac{1}{2}\right) - 0\right)\right] \\
&= \frac{1}{2} \times 1 \\
&= \frac{1}{2}.
\end{aligned}$$

We still have to solve for c:

$$\begin{aligned}
1 - |c| = f(c) &= \frac{1}{2} \\
\frac{1}{2} &= |c| .
\end{aligned}$$

Hence $c = \pm 1/2$.

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41. Use the formula for the future value of an object.

(a) After 20 days,

$$\begin{aligned} N(20) &= N(0) + \int_0^{20} N'(t)\ dt \\ &= 1500 + \int_0^{20} 100e^{-0.25t}\,dt \\ &= 1500 - 400e^{-0.25t}\Big|_0^{20} \\ &= 1500 - \left(400e^{-5} - 400e^{0}\right) \\ &\approx 1897. \end{aligned}$$

After 40 days, we have nearly the same thing; only the 20 changes, so

$$N(40) = 1500 - \left(400e^{-10} - 400e^{0}\right) \approx 1900.$$

(b) For the population t at any time $t \geq 0$,

$$N(t) = 1500 - \left(400e^{-0.25t} - 400e^{0}\right) = 1900 - 400e^{-0.25t}.$$

65. (a) Using the formula for net change, we want

$$\begin{aligned} E(24) - E(0) &= \int_0^{24} 300 - 200\sin\frac{\pi t}{12}\,dt \\ &= 300t + \frac{2400}{\pi}\cos\frac{\pi t}{12}\Big|_0^{24} \\ &= \left(300\times 24 + \cancel{\frac{2400}{\pi}\times 1}\right) - \left(300\times 0 + \cancel{\frac{2400}{\pi}\times 1}\right) \\ &= 7200\text{ MWh}. \end{aligned}$$

(b) One day needs $(7200\text{ MWh}) \div (450\ {}^{\text{kWh}}/_{\text{kg}}) = (7.2\times 10^9\text{ Wh}) \div (4.5\times 10^5\ {}^{\text{Wh}}/_{\text{kg}}) = 1.6\times 10^4$ kg, or 1600 kg for 1 day. Multiply by 365 to get the quantity needed for 1 year.

(c) One day needs $(7200\text{ MWh}) \div (16000\ {}^{\text{kWh}}/_{\text{g}}) = (7.2\times 10^9\text{ Wh}) \div \left(1.6\times 10^7\ {}^{\text{Wh}}/_{g}\right) = 4.5\times 10^2$ g, or 450 g for 1 day. Multiply by 365 to get the quantity needed for 1 year.

(d) If we assume that the wind turbines blow unceasingly each hour (a very optimistic assumption, incidentally), the city needs $(7200\text{ MWh}) \div (200\times 24\text{ kWh}) = (7.2\times 10^9\text{ Wh}) \div (4.8\times 10^6\text{ Wh}) = 1.5\times 10^3$, or 1500 turbines.