HOMEWORK QUIZ 1 SOLUTIONS

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13. The displacement is approximately

$$\begin{aligned} v(0)\cdot 2 + v(2)\cdot 2 + v(4)\cdot 2 + v(6)\cdot 2 &= 1\cdot 2 + \frac{1}{5}\cdot 2 + \frac{1}{9}\cdot 2 + \frac{1}{13}\cdot 2 \\ &= \frac{1624}{585} \\ &\approx 2.7761. \end{aligned}$$

35. The Left Riemann Sum is

$$5 \times .5 + 3 \times .5 + 2 \times .5 + 1 \times .5 = 5.5$$

while the Right Riemann Sum is

$$3 \times .5 + 2 \times .5 + 1 \times .5 + 1 \times .5 = 3.5.$$

67. (a) The object accelerates over 1 sec to 20 m/s, continues at that speed for 2 sec, decelerates for 2 sec to 10 m/s, and continues at that speed for at least 1 sec.
(b) $1/2 \times 1 \times 20 + 1 \times 20 = 30$
(c) $1 \times 20 + 1/2 \times 2 \times (20 + 10) = 50$
(d) The object's displacement over the first two periods is 80. That gets us to $t = 5$; thenceforth the particle proceeds at 10 m/s, whence its displacement is $80 + 10(t - 5)$. (We have to subtract 5 in order to determine how much time *after* 5 seconds has passed.) The book simplifies that expression to $30 + 10t$.

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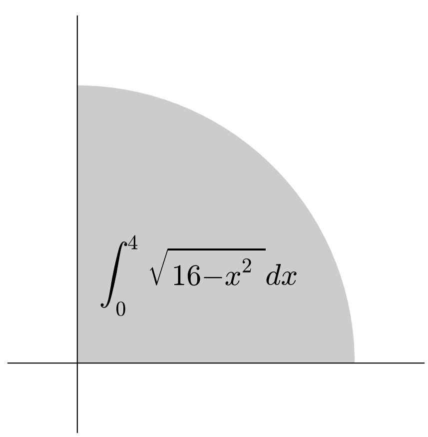
7. A figure with no width is one-dimensional, so there can be no area.

29. A sketch of the figure should look like this:



As the figure is a quarter circle, its area is $1/4 \times \pi \times 4^2 = 4\pi$.

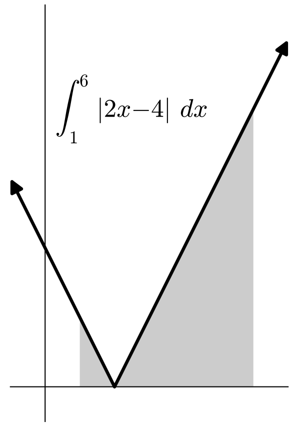
39. $\int_0^{2\pi} x \sin x \, dx = \cancel{1} + (\cancel{\pi} - \cancel{1}) - (\cancel{\pi} + \cancel{1}) - (2\pi - \cancel{1}) = -2\pi$

43. (a) $5 \int_0^3 f(x) \, dx = 5 \times 2 = 10$
(b) $-3 \int_3^6 g(x) \, dx = -3 \times 1 = -3$
(c) $3 \int_3^6 f(x) \, dx - \int_3^6 g(x) \, dx = 3(-5) - 1 = -16$
(d) $-\int_3^6 f(x) \, dx - 2 \int_3^6 g(x) \, dx = -(-5) - 2(1) = 3$

51. We have $a = 1$ and $b = 4$, so $\Delta x = 3/n$. For right endpoints we use $x_i^* = a + i\Delta x = 1 + 3i/n$. Now apply the definition of the integral:

$$\begin{aligned}
\int_1^4 \left(x^2 - 1\right) dx &= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(1 + \frac{3i}{n}\right)^2 - 1\right] \cdot \frac{3}{n} \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(\cancel{1} + \frac{6i}{n} + \frac{9i^2}{n^2}\right) - \cancel{1}\right] \cdot \frac{3}{n} \\
&= \lim_{n\to\infty} \left[\frac{3}{n}\left(\frac{6}{n}\sum_{i=1}^{n} i + \frac{9}{n^2}\sum_{i=1}^{n} i^2\right)\right] \\
&= \lim_{n\to\infty} \left[\frac{3}{n}\left(\frac{6}{\cancel{n}} \cdot \frac{\cancel{n}(n+1)}{2} + \frac{9}{n^{\cancel{2}}} \cdot \frac{\cancel{n}(n+1)(2n+1)}{6}\right)\right] \\
&= \frac{3\cdot 6}{2} \cdot \lim_{n\to\infty} \frac{n+1}{n} + \frac{3\cdot 9}{6} \cdot \lim_{n\to\infty} \frac{(n+1)(2n+1)}{n^2} \\
&= 9 \cdot \lim_{n\to\infty} \frac{n+1}{n} + \frac{9}{2} \cdot \lim_{n\to\infty} \frac{2n^2 + 3n + 1}{n^2} \\
&\overset{\text{L'H}}{=} 9 \cdot \lim_{n\to\infty} \frac{1}{1} + \frac{9}{2} \cdot \lim_{n\to\infty} \frac{4n+3}{2n} \\
&\overset{\text{L'H}}{=} 9 \cdot 1 + \frac{9}{2} \cdot \lim_{n\to\infty} \frac{4}{2} \\
&\overset{\text{L'H}}{=} 9 \cdot 1 + \frac{9}{2} \cdot 2 \\
&= 9 + 9 \\
&= 18.
\end{aligned}$$

73. A sketch of the figure should look like this:



The area is thus $1/2 \times 1 \times 2 + 1/2 \times 4 \times 8 = 1 + 16 = 17$.