MAT 167 TEST 1 FORM B (LIMITS)

Directions: Solve each required problem **on a separate sheet of paper. Use pencil and show all work;** I deduct points for using pen or skipping important steps. **You must shut off your cell phone.** Some problems are worth more than others. Take your time; **quality is preferred to quantity.** I encourage you to ask questions.

1. Give (a) an intuitive definition of continuity of a function f at a point $x = a$, and (b) the precise definition of continuity of a function f at a point $x = a$.

2. Sketch the graph of a function with the given properties. You need not find a formula for the function.

   $\lim_{x\to -\infty} f(x) = \infty$ $\quad f(-2) = 5 \quad f(0) = 0 \quad f(2) = 3$

   $\lim_{x\to\infty} f(x) = 3 \quad \lim_{x\to-2} f(x) = 5 \quad \lim_{x\to0^-} f(x) = -1 \quad \lim_{x\to2^-} f(x) = \infty$

   $\lim_{x\to0^+} f(x) = -1 \quad \lim_{x\to2^+} f(x) = -\infty$

3. Evaluate the following limits, if they exist. If they do not exist, state this. Justify your answers. One of them will need the Squeeze Theorem.

   (a) $\lim_{x\to -2} \frac{x+2}{x^2-4}$ (b) $\lim_{x\to1} f(x)$, where
   $$f(x) = \begin{cases} 1-x, & x<1 \\ \sqrt{x-1}, & x>1 \end{cases}$$
   (c) $\lim_{x\to\infty} \frac{\sin x}{x^3}$

   (d) $\lim_{x\to-2} \frac{x-2}{x^2-4}$ (e) $\lim_{x\to\infty} \frac{9x^2-1}{3x+1}$ (f) $\lim_{x\to\infty} \frac{3x^2+x+1}{3x^2+1}$

4. If possible, determine a value of b such that $p(x)$ is continuous at $x = -2$. If this is not possible, explain why not.
   $$p(x) = \begin{cases} 1-3x & \text{if } x<-2 \\ b & \text{if } x=-2 \\ x^2+3 & \text{if } x>-2. \end{cases}$$

5. Use the Intermediate Value Theorem to show that the equation $x^2-2=0$ has a solution on the interval $(1,2)$. Attempt neither to find nor to approximate the solution.

6. True or false? Explain why or why not.
   (a) When all three of them exist, $\lim_{x\to a} f(a)$ always equals both $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$.
   (b) The line $x = 2$ is a vertical asymptote of the function $f(x) = x^2-4/x-2$.