MAT 167 TEST 1 FORM A (LIMITS)

Directions: Solve each required problem **on a separate sheet of paper. Use pencil and show all work;** I deduct points for using pen or skipping important steps. **You must shut off your cell phone.** Some problems are worth more than others. Take your time; **quality is preferred to quantity.** I encourage you to ask questions.

1. Give (a) an intuitive definition of $\lim_{x\to a^-} f(x)$ (a one-sided limit), and

   (b) a precise definition of $\lim_{x\to a} f(x)$ (a two-sided limit).

2. Sketch the graph of a function with *all* of the given properties. You need not find a formula for the function.

$$\lim_{x\to -\infty} f(x)=\infty \qquad f(-2)=3 \qquad f(0) \text{ is undefined} \qquad f(2)=-3$$

$$\lim_{x\to\infty} f(x)=3 \qquad \lim_{x\to -2} f(x)=5 \qquad \lim_{x\to 0^-} f(x)=-\infty \qquad \lim_{x\to 2^-} f(x)=-3$$

$$\lim_{x\to 0^+} f(x)=-\infty \qquad \lim_{x\to 2^+} f(x)=-2$$

3. Evaluate the following limits, if they exist. If they do not exist, state this. Justify your answers. One of them will need the Squeeze Theorem.

   (a) $\displaystyle\lim_{x\to 3}\frac{x-3}{x^2-9}$

   (b) $\displaystyle\lim_{x\to 0} f(x)$, where $f(x)=\begin{cases} x^2-1, & x<0\\ 1-x^2, & x>0\end{cases}$

   (c) $\displaystyle\lim_{x\to\infty}\frac{x+1}{x^2-9}$

   (d) $\displaystyle\lim_{x\to -3}\frac{x-3}{x^2+6x+9}$

   (e) $\displaystyle\lim_{x\to\infty}\frac{\cos x}{x}$

   (f) $\displaystyle\lim_{x\to\infty}\frac{6x^2+x+1}{3x^2+1}$

4. If possible, determine a value of b such that $p(x)$ is continuous at $x=2$. If this is not possible, explain why not.

$$p(x)=\begin{cases} x+2 & \text{if } x<2\\ b & \text{if } x=2\\ x^2-2 & \text{if } x>2.\end{cases}$$

5. Use the Intermediate Value Theorem to show that the equation $\cos x - x = 0$ has a solution on the interval $\left(0,\frac{\pi}{2}\right)$. Attempt neither to find nor to approximate the solution.

6. True or false? Explain why or why not.

   (a) When $\lim_{x\to a} f(x)$ exists, it always equals $f(a)$.

   (b) The line $x=2$ is a vertical asymptote of the function $f(x)=x^2+4/x-2$.