QUIZ 1 SOLUTIONS

MAT 167

p. 65 #2. True or false: When $\lim_{x\to a} f(x)$ exists, it always equals $f(a)$. Explain.

False. It can happen that $f(a)$ doesn't exist, or that its value differs from the limit.

p. 67 #25. **Strange behavior near $\mathbf{x = 0}$**

(a) Create a table of values of $\sin(1/x)$, for $x = 2/\pi, 2/3\pi, \ldots, 2/11\pi$. Describe the pattern of values you observe.

The corresponding y-values are $1, -1, 1, -1, 1, -1$. The pattern alternates ± 1.

(b) Why does a graphing utility have difficulty plotting the graph of $y = \sin(1/x)$ near $x = 0$ (see figure *[omitted]*)?

As x approaches 0, $\sin(1/x)$ oscillates between 1 and -1 infinitely many times, as the pattern suggests, as well as each intermediate value. A graphing calculator cannot capture this infinite oscillation with finitely many points.

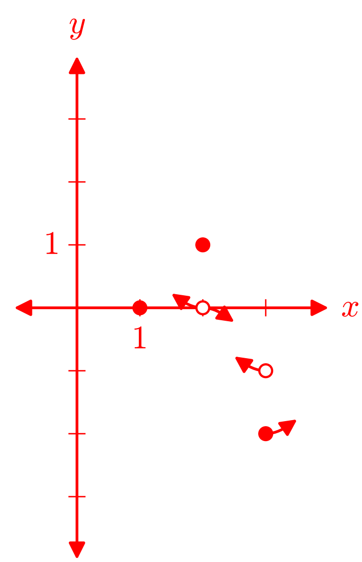
(c) What do you conclude about $\lim_{x\to 0} \sin\left(\frac{1}{x}\right)$?

The limit does not exist.

p. 68 #29. Sketch the graph of a function with the given properties. You do not need to find a formula for the function.

$g(1) = 0,\ g(2) = 1,\ g(3) = -2,\ \lim_{x\to 2} g(x) = 0,\ \lim_{x\to 3^-} g(x) = -1,\ \lim_{x\to 3^+} g(x) = -2$

As a minimum, I'd need to see the following features.



Notice that the axes are labeled and scaled. The arrows can point in a different vertical direction, but not in a different horizontal direction. They can extend further.

Date: Fall 2016.

p. 68 #35. **The floor function** For any real number x, the *floor function* (or *greatest integer function*) $\lfloor x \rfloor$ is the greatest integer less than or equal to x (see figure *[omitted]*).
Please note that $|x|$ and $\lfloor x \rfloor$ are two very different things. If you write the wrong one, it's wrong, period, full stop. I can't grade what's in your mind; I can only grade what's on your paper.

(a) Compute $\lim_{x\to -1^-} \lfloor x \rfloor$, $\lim_{x\to -1^+} \lfloor x \rfloor$, $\lim_{x\to 2^-} \lfloor x \rfloor$, and $\lim_{x\to 2^+} \lfloor x \rfloor$.
-2, -1, 1, and 2, in that order.
(You can verify this with a table of x- and y-values if you need to.)

(b) Compute $\lim_{x\to 2.3^-} \lfloor x \rfloor$, $\lim_{x\to 2.3^+} \lfloor x \rfloor$, and $\lim_{x\to 2.3} \lfloor x \rfloor$.
2, 2, and 2, in that order. (heh)
(Again, you can verify this with a table of x- and y-values if you need to.)

(c) For a given integer a, state the values of $\lim_{x\to a^-} \lfloor x \rfloor$ and $\lim_{x\to a^+} \lfloor x \rfloor$.
$a-1$ and a, in that order.
(To the left of $x = a$, the x-values are less than a, so the closest they can come is $a-1$, which they do very close to $x = a$. To the right of $x = a$, the x-values are larger than a, so the closest they can come is a itself, which they do at $x = a$.)

(d) In general, if a is not an integer, state the values of $\lim_{x\to a^-} \lfloor x \rfloor$ and $\lim_{x\to a^+} \lfloor x \rfloor$.
$\lfloor a \rfloor$ and $\lfloor a \rfloor$, in that order. (heh)
*(**Note:** The answer is not a. The y-values to the left and right of $a = 2.3$, for instance, are all 2, which means the limit is $2 = \lfloor 2.3 \rfloor = \lfloor a \rfloor$, **not** $a = 2.3$, as those who wrote "a" suggested implicitly.)*

(e) For what values of a does $\lim_{x\to a} \lfloor x \rfloor$ exist? Explain.
The limit exists for real numbers that are not integers; in short, $\mathbb{R}\setminus\mathbb{Z}$. These are the points described in part (d); their left- and right-sided limits agree with the value $\lfloor a \rfloor$. As we saw in part (c), this is not true of the integers, which have different left- and right-handed limits, which we can see by the fact that the graph jumps.