MAT 102 TEST 2, FORM A

Most solutions are for Form A. Solutions for Forms B and C are usually obtained by modifying the numbers appropriately. When this is not the case, or is not so obvious, I have noted the different answer for Forms B and C.

If you think you find an error, let me know. If you are right, I will give you extra credit.

1. (a) Give the precise definition of a **critical point**.

   A critical point is any x value where the derivative is zero or undefined.

   (Form B) Give the precise definition of an **inflection point**.

   An inflection point is any x value where concavity changes from up to down, or vice-versa.

   (Form C) Give the precise statement of the **zero product rule**.

   If a product is zero ($uv = 0$), then one of its factors must be zero ($u = 0$ or $v = 0$).

   (b) Does a function always change from increasing to decreasing, or vice-versa, at every critical point? Why or why not?
   A function does not always change from increasing to decreasing, or vice-versa at every critical point. Some functions increase, level off at a critical point, then increase again.

   (Form B) If concavity is negative from $x = a$ to $x = b$, how does that affect the slopes of the tangent lines at $x = a$ and $x = b$?

   The slopes are decreasing. The slope of the tangent line is the derivative, and concavity is the second derivative. When the second derivative is negative, the first derivative decreases.

   (Form C) Sometimes, students see an equation $x(x-2) = 1$ and try to solve it by setting $x = 1$ and $x - 2 = 1$. *This is an error!* Why?

   The second equation states $x = 1$ and the third implies $x = 3$, but neither solves the original: $1(1-2) = -1 \neq 1$ and $3(3-2) = 3 \neq 1$. The student is thinking of the zero product rule, but the product is not zero. There are many ways a product can equal 1, so you cannot assume that $uv = 1$ implies $u = 1$ and $v = 1$.

2. Let $f(x) = {}^{2x^2}/_{x^2-4}$.
   (a) Find the horizontal asymptote(s) of f, if any.
   Compare the degrees of numerator and denominator. Since they are equal (both 2) we determine the horizontal asymptote by dividing the leading coefficients of numerator and denominator. Thus, $y = {}^2/_1$ is the horizontal asymptote.

(b) Find the vertical asymptote(s) of f, if any.
We set the denominator to zero, and solve for x. The denominator is zero if $x^2-4=0$. Factoring gives us $(x-2)(x+2)=0$. By the zero product rule, $x-2=0$ or $x+2=0$. Solving for x tells us $x=2$ or $x=-2$. So, the vertical asymptotes are $x=\pm 2$.

(c) Use a sign diagram to identify the regions where f is increasing and the regions where f is decreasing.
We need the derivative:

$$f'(x)=\frac{(4x)\left(x^2-4\right)-\left(2x^2\right)(2x-0)}{(x^2-4)^2}=\frac{4x^3-16x-4x^3}{(x^2-4)^2}=\frac{-16x}{(x^2-4)^2}.$$

Set it to zero, and we find that

$$\frac{-16x}{(x^2-4)^2}=0$$
$$\left[\left(x^2-4\right)^2\right]\cdot\frac{-16x}{(x^2-4)^2}=0\left[\left(x^2-4\right)^2\right]$$
$$-16x=0$$
$$x=0.$$

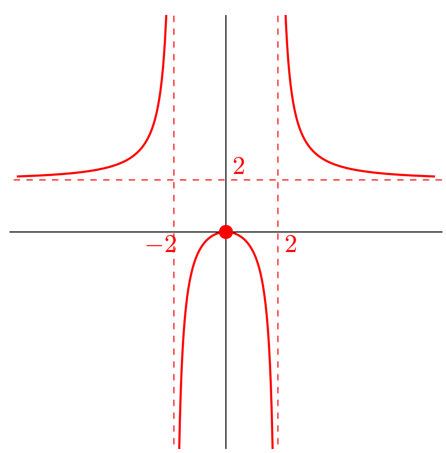
We have found only one critical point, $x=0$. The sign diagram must check around the critical point *and the vertical asymptotes:*

$f'(x)$	$+$	und	$+$	0	$-$	und	$-$
x	-3	-2	-1	0	1	2	3

The function is increasing on $(-\infty,-2)\cup(-2,-1)$ and decreasing on $(0,2)\cup(2,\infty)$.

(d) Identify the relative maxima and relative minima.
From the sign diagram we infer there is a relative maximum *at* $x=0$. Substituting this into the original function, we find the relative maximum *is* $f(0)={}^{2(0)}/_{(0^2-4)}={}^{0}/_{-4}=0$.

(e) Use the results of this problem to sketch a graph of f. Be sure to label all critical points and asymptotes.



3. Let $f(x)=2x^3-12x^2-13$.

(a) Use a sign diagram to identify the regions where f is increasing and the regions where f is decreasing.

We need the derivative: $f'(x) = 6x^2 - 24x$. Set it to zero to find the critical points:

$$6x^2 - 24x = 0 \implies 6x(x-4) = 0 \implies x = 0, 4.$$

There are no vertical asymptotes, so the sign diagram must check around the critical points only:

$f'(x)$	pos	0	neg	0	pos
x	-1	0	1	4	5

The function is increasing on $(-\infty, 0) \cup (4, \infty)$ and decreasing on $(0, 4)$.

(b) Use a sign diagram to identify the regions where f is concave up and the regions where f is concave down.
We need the second derivative: $f''(x) = 12x - 24$. Set it to zero to find the possible inflection points:

$$12x - 24 = 0 \implies x = 2.$$

The sign diagram must check around the potential inflection point only:

$f''(x)$	neg	0	pos
x	0	2	3

The function is concave up on $(2, \infty)$ and concave down on $(-\infty, 2)$.

(c) Identify the relative maxima and relative minima.
The sign diagram tells us that there is a relative maximum *at* $x = 0$ and a relative minimum *at* $x = 4$. Substituting this into the original function, we find the relative maximum *is* $f(0) = -13$ and the relative minimum *is* $f(4) = -77$.

(d) Use the results of this problem to sketch a graph of f. Be sure to label all critical points and inflection points.