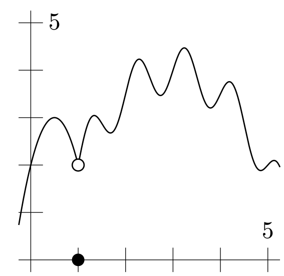
MAT 102 TEST 1, FORM B

Directions: Solve these problems on your own paper. Problems are not weighted equally, because some parts take more work than others. You may write on this paper, but I will not read it. Show all necessary work: **computations that are not obvious must be shown**. As to what is "obvious", better safe than sorry!

1. Give both a geometric definition and the precise definition of **continuity**.
   Geometrically, continuity means that we can draw the graph of a function without lifting our pencil at a hole, jump, or asymptote. Precisely, a function is continuous whenever we can evaluate the limit by substitution; that is, $\lim_{x\to a} f(x) = f(a)$.

2. Given the graph of $f(x)$ at right,
   (a) find $\lim_{x\to 1^-} f(x)$; 2
   (b) find $\lim_{x\to 1^+} f(x)$; 2
   (c) find $\lim_{x\to 1} f(x)$. 2
   (d) Is f continuous at $x = 1$? no



3. Compute $\lim_{x\to 4} \frac{x^2-16}{x-4}$ using a table of values.

x	y
3	7
3.5	7.5
3.9	7.9
3.99	7.99
4	?
4.01	8.01
4.1	8.1
4.5	8.5
5	9

It looks as if the limit is 8.

4. Let $f(x) = 2 - x^3$.
   (a) Find the average rate of change between $x = 1$ and $x = 2$.

$$\frac{\Delta y}{\Delta x} = \frac{f(2) - f(1)}{2-1} = \frac{(2-2^3)-(2-1^3)}{1} = \frac{-6-1}{1} = -7$$

(b) Find $f'(x)$ using the ***definition*** of the derivative.

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\left[2-(x+h)^3\right]-\left(2-x^3\right)}{h}$$
$$=\lim_{h\to 0}\frac{\left[\cancel{2}-\left(\cancel{x^3}+3x^2h+3xh^2+h^3\right)\right]-\left(\cancel{2}-\cancel{x^3}\right)}{h}$$
$$=\lim_{h\to 0}\frac{-\cancel{h}\left(3x^2+3xh+h^2\right)}{\cancel{h}}$$
$$=-\left(3\cdot x^2+3x\cdot 0+0^2\right)$$
$$=-3x^2.$$

(c) Use your answer to part (b) to find the instantaneous rate of change at $x=1$.
By substitution, $f'(1)=-3\cdot 1^2=-3$.

5. Use the ***properties*** of the derivative to evaluate the derivative of each of the following functions.

(a) $f(x)=2-x^3$ (b) $f(x)=\left(2-x^3\right)\left(2x^2-x\right)$

(c) $f(x)=\dfrac{2-x^3}{2x^2-x}$ (d) $f(x)=3\left(2-x^3\right)^{20}-2$

(a) is straightforward: $f'(x)=0-3x^2=-3x^2$.
(b) can be attacked by expansion or the product rule, which gives $-3x^2\left(2x^2-x\right)+\left(2-x^3\right)\cdot(4x-1)$.
(c) requires the quotient rule:

$$f'(x)=\frac{\underbrace{-3x^2}_{\text{deriv first}}\underbrace{\left(2x^2-x\right)}_{\text{second}}-\underbrace{\left(2-x^3\right)}_{\text{first}}\cdot\underbrace{(4x-1)}_{\text{deriv second}}}{\underbrace{\left(2x^2-x\right)^2}_{\text{second squared}}},$$

and you can stop there. In fact, you probably *should* stop there.
(d) requires the chain rule, because it has the form $y=3u^{20}-2$ where $u=2-x^3$. The chain rule is

$$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}=\left(3\cdot 20u^{19}-0\right)\cdot\left(-3x^2\right)=60u^{19}\cdot\left(-3x^2\right)=-180x^2\left(2-x^3\right)^{19}.$$

6. Suppose the number of traffic fatalities per hundred million miles traveled approximates

$$f(x)=\frac{2}{\sqrt{x}}+1,$$

where x stands for the number of years *since 1975*. Find the rate of change of this percentage in the year 2004, and interpret your answer.

The year 2004 occurs when $x = 29$ (years *since 1975*) and "rate of change" means "derivative". So,

$$f'(x) = 2 \cdot \left(-\frac{1}{2}x^{-3/2}\right) + 0 = -\frac{1}{x\sqrt{x}}.$$

Evaluated at $x = 29$, we find that

$$f'(29) = -\frac{1}{29\sqrt{29}} \approx -0.0064.$$

This means that the number of traffic fatalities was decreasing by roughly .6% in 2004.