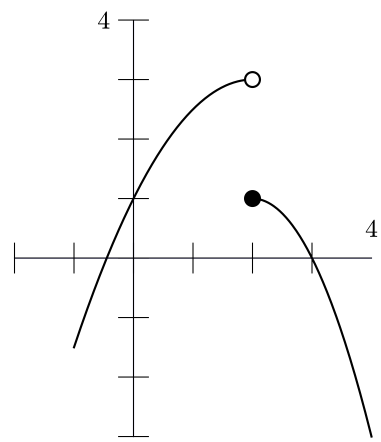
QUIZ SOLUTIONS

Quiz the first

1. Given the graph of $f(x)$ at right, find $\lim_{x\to 2^-} f(x)$, $\lim_{x\to 2^+} f(x)$, and $\lim_{x\to 2} f(x)$.

   Solution: $\lim_{x\to 2^-} f(x) = 3$, $\lim_{x\to 2^+} f(x) = 1$. Since the limits disagree, $\lim_{x\to 2} f(x)$ is undefined.

2. Evaluate $\lim_{x\to 0}\left(4x^2 - 3x + 4\right)$.

   Solution: Since $4x^2 - 3x + 4$ is a polynomial, and therefore continuous, we can evaluate the limit by substitution. Thus, $\lim_{x\to 0}\left(4x^2 - 3x + 4\right) = 4 \cdot 0^2 - 3 \cdot 0 + 4 = 4$.

3. Which function(s) is (are) continuous everywhere?

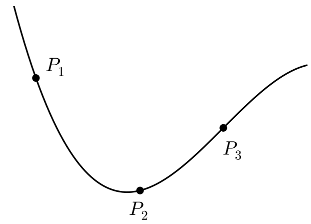
   $$4x^2 - 3x + 4 \qquad -12 \qquad \frac{3}{x} \qquad e^x$$

   Solution: Polynomials are continuous everywhere, so the first two functions are continuous. (Constants are also polynomials.) The third function has division by 0 at $x = 0$, so it cannot be continuous. Using the intuitive definition of continuity, we can determine that e^x is also continuous, because we never have to pick up the pencil while drawing its graph.

4. What was §2.3 about?

   Solution: Shortcuts for taking derivatives. — *Or,* derivative properties. — *Or,* Marginal Cost and its ilk.

QUIZ THE SECOND



1. By imagining tangent lines at P_1, P_2, and P_3 in the graph, state whether the slopes are positive, negative, or zero.
Solution: Slope is negative at P_1, zero at P_2, positive at P_3.

2. For $f(x) = x^2 - 2x$, find the average rate of change between the given points. Then, indicate the value the average rates of change are approaching.
(a) $x = 0, x = 2$ (b) $x = 0, x = 1$ (c) $x = 0, x = 1/2$ (d) $x = 0, x = 1/10$
Solution:

$$\text{(a) } \frac{\Delta y}{\Delta x} = \frac{f(2) - f(0)}{2 - 0} = \frac{(2^2 - 2\cdot 2) - (0^2 - 2\cdot 0)}{2} = \frac{0-0}{2} = 0$$

$$\text{(b) } \frac{\Delta y}{\Delta x} = \frac{f(1) - f(0)}{1 - 0} = \frac{(1^2 - 2\cdot 1) - (0^2 - 2\cdot 0)}{1} = \frac{-1-0}{1} = -1$$

$$\text{(c) } \frac{\Delta y}{\Delta x} = \frac{f(1/2) - f(0)}{1/2 - 0} = \frac{((1/2)^2 - 2\cdot 1/2) - (0^2 - 2\cdot 0)}{1/2} = \frac{-3/4-0}{1/2} = -1.5$$

$$\text{(d) } \frac{\Delta y}{\Delta x} = \frac{f(1/10) - f(0)}{1/10 - 0} = \frac{((1/10)^2 - 2\cdot 1/10) - (0^2 - 2\cdot 0)}{1/10} = \frac{19/100}{1/10} = -1.9$$

The values approach -2.

3. Use the definition of the instantaneous rate of change to find $f'(0)$, where $f(x) = x^2 - 2x$.
Solution: First we need to find $f'(x)$:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\overbrace{\left[(x+h)^2 - 2(x+h)\right]}^{f(x+h)} - \overbrace{(x^2 - 2x)}^{f(x)}}{h}$$

$$= \lim_{h\to 0} \frac{\left[(x^2 + 2xh + h^2) - 2(x+h)\right] - (x^2 - 2x)}{h} \qquad \text{expand } (x+h)^2$$

$$= \lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{- 2x} - 2h \cancel{- x^2} \cancel{+ 2x}}{h} \qquad \text{distribution}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2x + h - 2)}{\cancel{h}} \qquad \text{factoring}$$

$$= \lim_{h\to 0} (2x + h - 2)$$

$$= 2x + 0 - 2.$$

Since $f'(x) = 2x - 2$, we can find $f'(0) = 2\cdot 0 - 2 = -2$.

QUIZ THE THIRD

1. Find the derivative of $\frac{1}{64}x^8 - \frac{1}{16}x^4 + 2x + 1$.
Solution: Use the properties:

$$\frac{d}{dx}\left(\frac{1}{64}x^8 - \frac{1}{16}x^4 + 2x + 1\right) = \left(\frac{d}{dx}\frac{1}{64}x^8\right) - \left(\frac{d}{dx}\frac{1}{16}x^4\right) + \left(\frac{d}{dx}2x\right) + \left(\frac{d}{dx}1\right) \tag{1}$$

$$= \frac{1}{64}\left(\frac{d}{dx}x^8\right) - \frac{1}{16}\left(\frac{d}{dx}x^4\right) + 2\left(\frac{d}{dx}x\right) + \left(\frac{d}{dx}1\right) \tag{2}$$

$$= \frac{1}{64}\left(8x^7\right) - \frac{1}{16}\left(4x^3\right) + 2(1) + (0) \tag{3}$$

$$= \frac{1}{8}x^7 + \frac{1}{4}x^3 + 2. \tag{4}$$

We used
(1) the derivative of a sum is the sum of the terms' derivatives;
(2) the derivative of a constant multiple is the constant multiple of the derivative;
(3) the derivative of x^n is nx^{n-1}; the derivative of x is 1; and the derivative of 1 is 0;
(4) "obvious" simplifications.

2. If $f(x) = \frac{1}{64}x^8 - \frac{1}{16}x^4 + 2x + 1$, find $f'(2)$.
Solution: We already found the derivative in the first problem, so we need merely substitute $x = 2$:

$$f'(2) = \frac{1}{8}\cdot 2^7 - \frac{1}{4}\cdot 2^3 + 2 = 16 \cancel{-2+2} = 16.$$

3. A company sells widgets at a total cost of approximately $C(x) = 32x^{3/4}$ dollars for x widgets. Find the marginal cost of 10,000 widgets, and interpret your answer.
Solution: Marginal cost is approximately equal to the derivative, so we compute

$$MC(x) = \frac{d}{dx}\left(32x^{3/4}\right) = 32\frac{d}{dx}\left(x^{3/4}\right) = 32\cdot\frac{3}{4}x^{3/4-1} = 24x^{-1/4} = \frac{24}{x^{1/4}}.$$

When $x = 10,000 = 10^4$, we have

$$MC\left(10^4\right) = \frac{24}{\left(10^4\right)^{1/4}} = \frac{24}{10} = 2.4.$$

4. Use the *intuitive* definition of the derivative to explain why the derivative of the function $f(x) = -3x$ must be $f'(x) = -3$.
(*Hint:* By "intuitive", I refer to the geometric interpretation of the derivative.)
Solution: The function $f(x) = -3x$ is a line. For any x, the line tangent to f at x will coincide with f, because the only line tangent to a line is the line itself. The derivative is the slope of the tangent line, hence the slope of f, which is -3.

5. A company can produce LCD digital alarm clocks at a cost of $C(x) = 3x - 1000$ dollars for x clocks.
   (a) Find the average cost function, $AC(x)$.
   Solution: We find the average by dividing a total by the number of objects involved in computing that total, so $AC(x) = {}^{C(x)}/_{x} = {}^{3x-1000}/_{x}$. You *can* simplify this to $AC(x) = 3 - {}^{1000}/_{x}$ or even $AC(x) = 3 - 1000x^{-1}$, but you don't need to.
   (b) Find the marginal average cost function, $MAC(x)$.
   Solution: The second formulation makes it easiest to compute the derivative, since

$$MAC(x) = AC'(x) = 0 - 1000\left(-x^{-2}\right) = \frac{1000}{x^2}.$$

   The first formulation requires the quotient rule:

$$MAC(x) = AC'(x) = \frac{(3-0)\cdot x - (3x-1000)\cdot 1}{x^2} = \frac{3x-(3x-1000)}{x^2} = \frac{1000}{x^2}.$$

   (c) Evaluate $MAC(x)$ at $x = 350$, rounded to the nearest tenth of a cent.
   Solution: $MAC(350) = {}^{1000}/_{x^2} = 0.008$. (When measuring in dollars, "cents" appear in the second decimal place, so "tenths of a cent" appear in the third.)