LINEAR PROGRAMMING PROJECT

The following is to be done on a Sage worksheet. Anything mentioned as "commentary" or "explain" should be typed in an HTML box, not in a computational box. Please use LaTeX when appropriate; see the list of useful formatting directives on the back. Also, please label each problem by placing its number in an HTML box, and changing its formatting from Paragraph to Heading 2. Likewise, label each of Part 1 and Part 2 by placing it in a box before its exercises, and changing its formatting from Paragraph to Heading 1.

This is due no later than Friday, May 7th, at noon. Earlier is better, though.

1. MIXED INTEGER PROGRAMMING

In this project, you will work with the linear program

$$\begin{aligned} &\text{Minimize } x+y \text{ subject to} \\ &-10x+y \le -.01 \\ &-5x+y \ge .01 \\ &x,y \ge 0 \end{aligned}$$

(1) Solve the LP relaxation using Sage's `MixedIntegerLinearProgram` facility.
(2) Use the solution to #1 to plot the feasible region. (Hint: use the `polygon()` function with the solution found in #1.)
(3) Solve the integer program using Sage's `MixedIntegerLinearProgram` functions by setting the variables to be integer variables.
(4) Suppose we want an integer solution for x, but y can be real. Find the optimal solution in this case.
(5) Now that you have these solutions as given by Sage, perform a step-by-step solution of the LP relaxation using the compact tableau. Use Sage to manipulate a matrix that represents the tableau. You may use the pivoting program that I have provided in previous worksheets.
(6) Now you that have solved the LP relaxation using the compact tableau, solve the pure integer program using Gomory cuts. Again, use Sage to manipulate a matrix that represents the modified tableau.
(7) Illustrate the cutting method, adding to the plot from #2 dashed line(s) to illustrate the Gomory cut(s) used. (Hint: use the `plot()` function with option `linestyle='--'`.)

2. "IN"SENSITITIVY ANALYSIS

In the chapter on sensitivity analysis, we showed how the compact tableau had to change if we changed the objective function, a constant term of the constraints, added a constraint, or added a variable.

Suppose we have a feasible solution, and we *delete* a constraint. Do we have to restart the simplex algorithm in this case as well? We will explore this in two dimensions; that is, we have only x and y.

(1) Modify the linear program given at the beginning by adding a constraint that does *not* change the optimal value. (Hint: think geometrically.) Describe how the compact tableau changes. Show that it does not change the optimal value by applying the dual simplex algorithm. Illustrate this using a graph, and commentary.
(2) Modify the result of #1 by removing the new constraint. Describe how the compact tableau changes.
(3) Modify the linear program given at the beginning by adding a constraint that *does* change the optimal value. Describe how the compact tableau changes. Show that it changes by applying the dual simplex algorithm. Illustrate this using a graph, and commentary.
(4) What do we look for in the final tableau of #3 to help us decide which row or column gives us information about the ith constraint?
(5) Use your answer to #4 to suggest a method for removing the ith constraint from the compact tableau of a linear program.

3. Useful LaTeX functions for commentary in Sage

Remember to enclose these commands in dollar signs, e.g. $x \in \mathbb{R}$

LaTeX notation	**concept**	**example in LaTeX**	**result**
{...}	grouping	see below	see below
\mathrm{...}	don't italicize ...	\mathrm{next}	next
\mathbb{...}	write ... in "blackboard bold"	\mathbb{R}	$\mathbb{R}$
\mathbf{...}	write ... in bold font	a\mathbf{F}	$a\mathbf{F}$
\mathcal{...}	write ... in calligraphic font	\mathcal{S}	$\mathcal{S}$
^	superscript	x^2	x^2
_	subscript	x_{\mathrm{next}}	x_{next}
\in	element of	x\in S	$x \in S$
\{...\}	a set containing ...	\{1,5,7\}	$\{1,5,7\}$
\frac{*a*}{*b*}	fraction of *a* over *b*	\frac{2}{5}	$\frac{2}{5}$
\alpha, \beta, etc.	Greek letters	2\pi	2π
\infty	infinity	(-\infty,\infty)	$(-\infty,\infty)$
\sum	summation symbol	\sum_{i=1}^n a_{ij}x_j	$\sum_{i=1}^n a_{ij}x_j$
\leq, \geq	$\leq, \geq$	a\leq b	$a \leq b$
\notin, \neq	$\notin, \neq$	a\not\in S	$a \not\in S$
\subset, \not\subset	$\subset, \not\subset$	S\not\subset T	$S \not\subset T$
\ldots, \cdots, \ddots	$\ldots, \cdots, \ddots$	\mathbb{N}=\{1,2,\ldots\}	$\mathbb{N}=\{1,2,\ldots\}$
\cap, \cup	intersection, union	S\cap(T\cup U)	$S\cap(T\cup U)$