MAT 418 Handout: Day 1

January 18, 2012

1 Minimal Sage

The department's Sage server is located at

`https://pax.st.usm.edu:8004/`

Sometimes the software has to be updated, and that requires a machine reset, which means Sage goes offline. If you want it brought back online before I notice it's out, email me.

Sage computations are organized in cells (those boxes). You can type commands in a cell. If you press <TAB> at any point, Sage will try to complete the word you started with an object or command it recognizes. For example, if I type

`Mix<TAB>`

then Sage will complete the word

`MixedIntegerLinearProgram`

If Sage cannot decide which word you are trying to complete, it will pop up a box with a list of possible words. You can click on one, use arrows to navigate through the list, or just keep typing.

The basic flow for creating and solving a linear program is as follows:

1. `lp = MixedIntegerLinearProgram()`

   This gives you access to an LP solver that will maximize *objective* functions. You can use any legitimate identifier instead of `lp`, such as `p`, `my_program`, `cats_will_cause_the_mayan_apocalypse`, and so forth.

   If you want a solver that *minimizes* a linear program, do this instead:

   `lp = MixedIntegerLinearProgram(maximization=False)`

2. `x = lp[1]; y = lp[2]; ...`

   This creates variables that the LP solver will recognize. You can omit the semicolons if you place them on separate lines. You can also use `lp[1]`, `lp[2]`, and so forth, *without* defining variables. That can be useful, but for now I'd prefer you define variables.

3. `lp.add_constraint(x + 2*y >= 3); lp.add_constraint(2*x - y <= 2); ...`

   This adds the constraint $x + 2y \geq 3$ to the LP. Notice that **you must type an asterisk** to show multiplication; if you do not, Sage will complain.

4. `lp.set_objective(x + y)`

   This sets the objective function (the function to maximize) as $z = x + y$. We want the largest possible value of z.

5. `lp.solve()`

   Now that we have defined constraints and an objective function, we can try to solve the LP. *If* there is a solution, this will also tell you what the maximum (or minimum) value of the objective function is. The LP we've defined to this point has no solution, and reports it in this rather terrifying fashion:

```
---------------------------------------------------------------------------
MIPSolverException Traceback (most recent call last)
/atlas/perry/Downloads/sage-4.8.rc0/devel/sage-main/<ipython console>
in <module>()
/atlas/perry/Downloads/sage-4.8.rc0/local/lib/python2.6/site-packages/sage/numeri
in sage.numerical.mip.MixedIntegerLinearProgram.solve (sage/numerical/mip.c:6863)
/atlas/perry/Downloads/sage-4.8.rc0/local/lib/python2.6/site-packages/sage/numeri
in sage.numerical.backends.glpk_backend.GLPKBackend.solve (sage/numerical/backend
MIPSolverException:  'GLPK : Solution is undefined'
```

   That's the way MILP is set up, I'm afraid.

6. `lp.get_values([lp[1], lp[2]])`

   If there is a solution, this will return the values of the variables that give the solution. If there is no solution, it will return answers nevertheless, **but they will be invalid answers.**

To see this in action with a problem that actually works, recreate the integer program from step 1, but change the constraint `x + 2*y >= 3` to `x + 2*y <= 3`. You should be able to see that the new LP has a solution of 2.2 with solutions $x = 1.4$ and $y = 0.8$.

2 Homework due Monday, 23 Jan 2012

1. Adeline, the cook, wants to mix a cheap granola with a more expensive variety to offer a nutritious breakfast to her dormitory residents. Each pound of Chintz contains 4 units of protein, 4 units of carbohydrate, and 24 units of fat. Each pound of Scrumch contains 12 units of protein, 4 units of carbohydrate, and 8 units of fat. A 50-pound bag of Chintz costs $25, and a 20-pound bag of Scrumch costs $16. Adeline feels that each student should have for breakfast at least 6 units of protein, 4 units of carbohydrates, and 12 units of fat.

   (a) Fill in the table with this information.

Granola	Protein	Carbohydrate	Fat	Cost per pound
Chintz				
Scrumch				
Units needed				

(b) Set up an LP problem to answer the question: How many pounds of Chintz and how many pounds of Scrumch should Adeline serve each student to provide the minimum dietary requirements at the least cost?

(c) Use Sage (or some other computer-based solver) to set up and solve the linear program.

2. The Mazda Corporation manufactures light bulbs by assembling three components: a base, a glass globe, and a filament. Mazda usually manufactures its own components, though the firm can purchase these components from outside sources when the quantities needed exceed Mazda's production capacity. Mazda has contracted to produce 12,000 bulbs this month. The table below gives the costs of manufacturing components inside and purchasing them outside.

Component	Inside cost ($)	Outside cost ($)
Base	0.05	0.06
Globe	0.03	0.04
Filament	0.10	0.14

Mazda's plant is organized into three departments. The table below gives the time requirements for manufacturing components. The last line of the table indicates Mazda's capacity (hours available for production).

	Department		
Component	Cutting (hr)	Shaping (hr)	Assembly (hr)
Base	0.04	0.05	0.06
Globe	0.07	0.03	0.05
Filament	0.06	0.03	0.06
Capacity	1600	1400	1500

(a) Set up an LP problem to answer the question: How many of each component should be manufactured inside and how many purchased outside to minimize production costs?

(b) Use Sag (or some other computer-based solver) to set up and solve the linear program.