**Algorithm 1** F5C

**globals** $r$, *Rule*, $<_T$<br>
**inputs**<br>
  $F = (f_1, f_2, \ldots, f_m) \in \mathcal{R}^m$ (homogeneous)<br>
  $<$, an admissible ordering<br>
**outputs**<br>
  a Gröbner basis of $F$ with respect to $<$<br>
**do**<br>
  $<_T := <$<br>
  Sort $F$ by increasing total degree, breaking ties by increasing leading monomial<br>
  — Initialize the record keeping.<br>
  $\mathit{Rule} := \mathrm{List}\left(\mathrm{List}\left(\right)\right)$<br>
  $r := \mathrm{List}\left(\right)$<br>
  Append $\left(\mathbf{F}_1, f_1 \cdot \mathrm{lc}\left(f_1\right)^{-1}\right)$ to $r$<br>
  — Compute the basis of $\langle f_1 \rangle$.<br>
  $G_{\mathrm{prev}} = \{1\}$<br>
  $B = \{f_1\}$<br>
  — Compute the bases of $\langle f_1, f_2 \rangle, \ldots, \langle f_1, f_2, \ldots, f_m \rangle$.<br>
  $i := 2$<br>
  **while** $i \leq m$<br>
    Append $\left(\mathbf{F}_{\#r+1}, f_i \cdot \mathrm{lc}\left(f_i\right)^{-1}\right)$ to $r$<br>
    $G_{\mathrm{curr}} :=$ Incremental_Basis (F5) $\left(\#r, B, G_{\mathrm{prev}}\right)$<br>
    **if** $\exists \lambda \in G_{\mathrm{curr}}$ such that $\mathrm{Poly}\left(\lambda\right) = 1$<br>
      **return** $\{1\}$<br>
    — The only changes from F5 are incorporated here<br>
    $G_{\mathrm{prev}} :=$ Setup_Reduced_Basis $\left(G_{\mathrm{curr}}\right)$<br>
    $B := \{\mathrm{Poly}\left(\lambda\right) : \lambda \in G_{\mathrm{prev}}\}$<br>
    $i := i + 1$<br>
  **return** $B$

**Algorithm 2** SETUP_REDUCED_BASIS

**globals** $r$, *Rule*
(modifies $r$ and *Rule*)
**inputs**
$G_{\text{prev}}$, a list of indices of polynomials in $r$ that correspond to a Gröbner basis of $(f_1, \ldots, f_i)$
**outputs**
$G_{\text{curr}} \subset \mathbb{N}$, indices of polynomials in $r$ that correspond to a *reduced* Gröbner basis of $(f_1, \ldots, f_i)$
**do**
Let $B$ be the reduced Gröbner basis of $\{\text{Poly}(k) : k \in G_i\}$
$G_{\text{curr}} := \{j\}_{j=1}^{\#B}$
$r := \text{List}\left(\{(\mathbf{F}_j, B_j)\}_{j=1}^{\#B}\right)$
— All the $S$-polynomials of $B$ reduce to zero; document this
$Rule = \text{List}\left(\{\text{List}\,()\}_{j=1}^{\#B}\right)$
**for** $j := 1$ **to** $\#B$
$t := \text{lt}(B_j)$
**for** $k := j + 1$ **to** $\#B$
$u := \text{lcm}(t, \text{lt}(B_k)) / \text{lt}(B_k)$
ADD_RULE $(u\mathbf{F}_k, 0)$
**return** $G_{\text{curr}}$